

Archival note: symmetric monodromy in a weighted-lift subfamily

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Archival status: main theorem subsumed by earlier, stronger work

ABSTRACT

Gallagher's weighted-lift construction, posted after Alpgoe's announced counterexample, produces a three-dimensional Keller map of every generic degree $n \geq 3$. We isolate a simple uniform subfamily and prove that its function-field extension has full symmetric Galois closure S_n , hence no nonidentity rational deck transformations. A uniform rational collision is also given. Mikhail Szh published the stronger monodromy theorem for the entire weighted-lift family before this note appeared. This archival version is retained for provenance; only the explicit seed and uniform collision survive as potentially new data.

1. The explicit family

Fix an integer $n \geq 3$ and put

$$u = 1 + xy, \quad \text{gamma} = 1 - [n/(n-1)]xy + x^2z.$$

Define $F_n = (A_n, B_n, C_n)$ by

$$A_n = \{(n-2)u + u^2 - (n-1)u^n \text{gamma}^{n-2}\} / \{(n-2)x^2\},$$
$$B_n = \{(n-2) + 2u - n u^{n-1} \text{gamma}^{n-2}\} / \{(n-2)x\}, \quad C_n = x \text{gamma}. \quad (1)$$

The apparent quotients by x^2 and x cancel identically, so (1) is a polynomial map over $\mathbb{Q}$.

Theorem 1. For every $n \geq 3$, the map F_n is polynomial and $\det JF_n = 1$; the extension $C(x,y,z)/C(F_n)$ has degree n and Galois closure S_n ; and every rational map σ satisfying $F_n \circ \sigma = F_n$ is the identity.

2. Polynomiality and constant Jacobian

Set $v = xy$, $\tau = x^2z$, and $w = u \text{gamma}$. Define

$$p_n(w) = [2w - n w^{n-1}]/(n-2), \quad q_n(w) = [w^2 - (n-1)w^n]/(n-2).$$

Then $q_n' = w p_n'$ and (1) is $(\alpha/x^2, \beta/x, x \text{gamma})$, where $\alpha = u + q_n(w)/\text{gamma}^2$ and $\beta = 1 + p_n(w)/\text{gamma}$. As polynomials in (v, τ) , β lies in (v, τ) and α lies in (v^2, τ) ; substitution proves the required divisibility.

Put $P = BC = \text{gamma} + p_n(w)$ and $Q = AC^2 = w \text{gamma} + q_n(w)$. The output change $(A, B, C) \rightarrow (P, Q, C)$ has Jacobian $-C^3$. The input changes through (x, w, gamma) and the identity $w p' - q' - \text{gamma} = -\text{gamma}$ give the same Jacobian $-C^3$. Therefore $\det JF_n = 1$.

3. Inverse equation and degree

With $R_n(w) = (w^2 - w^n)/(n-2)$, the identity $q_n = w p_n - R_n$ eliminates gamma and gives $R_n(w) = wP - Q$. Thus the fiber polynomial is

$$H_n(T) = T^n - T^2 + (n-2)BCT - (n-2)AC^2. \quad (2)$$

A root w with $\text{gamma} = P - p_n(w)$ nonzero recovers the source rationally: $x = C/\text{gamma}$, $u = w/\text{gamma}$, $y = (u-1)/x$, and $z = \{\text{gamma}-1 + [n/(n-1)](u-1)\}/x^2$.

Writing $U = (n-2)BC$ and $V = -(n-2)AC^2$, the target field is $K = C(A,B,C) = C(U,V,C) = C(U,V)(C)$, since $B = U/((n-2)C)$ and $A = -V/((n-2)C^2)$. Here C is transcendental over $C(U,V)$. Equation (2) is $h_n(T) = T^n - T^2 + UT + V$. It is irreducible over $C(U,V)$: being linear in V with coefficient one, any factor independent of V would be a unit. Gauss's lemma preserves irreducibility under the purely transcendental base change to K . Hence the extension degree is n .

4. Full symmetric monodromy

The discriminant curve of h_n is parametrized by

$$t \rightarrow (U,V) = (2t - n t^{n-1}, (n-1)t^n - t^2). \quad (3)$$

It is irreducible. At $t = 0$ the polynomial is $T^2(T^{n-2}-1)$, with exactly one double root. It is the unique parameter over $(U,V) = (0,0)$, and the derivative of (3) has first coordinate 2, so this is a smooth discriminant point. The one-double-root condition is nonempty and Zariski open on the irreducible curve, so generic inertia is a transposition. The root-incidence variety is A^2 , hence connected, so the monodromy action is transitive. Quotienting the Galois closure by the normal subgroup generated by finite inertia yields a cover of A^2 unramified in codimension one. Purity makes it finite etale, and A^2 has no nontrivial connected finite etale cover. Therefore the monodromy group is generated by conjugate transpositions. A transitive group generated by transpositions is S_n . Finally, a finite algebraic extension is linearly disjoint from the purely transcendental extension $C(U,V)(C)$, so this remains the Galois group over the actual target field K .

Corollary. The rational deck group is trivial. In the S_n -closure, the root field is fixed by a point stabilizer S_{n-1} , which is self-normalizing; hence $\text{Aut}_K(L) = N(S_{n-1})/S_{n-1} = 1$.

5. Uniform rational collision

Let $s_n = (4 - 2^n)/(n-2)$. At the target $(s_n, s_n, 1)$, the fiber polynomial is $T^n - T^2 + (4-2^n)(T-1)$, with simple roots 1 and 2. For r in $\{1,2\}$, set $g_r = s_n - p_n(r)$ and

$$X_{n,r} = (1/g_r, r-g_r, g_r^2[g_r-1 + n/(n-1)(r/g_r-1)]). \quad (4)$$

Here $g_1 = (n+2-2^n)/(n-2)$ and $g_2 = 2^{n-1}$, both nonzero. Exact substitution gives $F_n(X_{n,1}) = F_n(X_{n,2}) = (s_n, s_n, 1)$. For $n = 3$: $F_3(-1/3, 4, -54) = F_3(1/4, -2, 36) = (-4, -4, 1)$.

6. Corrected status and reproducibility

Mikhail Szh's initial public commit at 04:03:36 UTC on 21 July 2026 proves full S_n monodromy for every valid weighted-lift seed. It predates this note's 13:11:39 UTC release and subsumes the monodromy and deck-group claims. The explicit seed (1) and uniform rational collision (4) are preserved as Appendix A of the canonical 22/44-variable paper. This note is archival and should not be cited as an independent discovery.

References

- [1] O.-H. Keller, *Ganze Cremona-Transformationen*, Monatsh. Math. Phys. 47 (1939), 299-306.
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- [3] L. Alpoge, announcement of an explicit counterexample, 19 July 2026.
- [4] A. Gallagher, *An infinite family of counterexamples to the Jacobian Conjecture in dimension three: every generic fiber degree $n \geq 3$ occurs*, 20 July 2026.
- [5] MathOverflow 513387, *Galois structure of the new counterexample to the Jacobian conjecture*, 20 July 2026.
- [6] Mikhail Szh and Claude Fable, *The Antiderivative Resolvent of the Weighted-Lift Family of Keller Maps*, initial public commit f8a6d6794febb551050d73b8cf6ffab9da52d047, 21 July 2026.

Disclosure: Alec Kriebel is a complete amateur and cannot independently verify the mathematical claims in this note. The formulas, proof organization, verifier, and typeset draft were developed with heavy assistance from ChatGPT 5.6 Sol. Independent expert review is required.