Excluding Parameter Three at Order Thirteen in the γ–θ Conjecture for One-Guard Eternal Domination

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Abstract

In the one-guard-moves eternal domination game, attacks are made only at unoccupied vertices and exactly one adjacent guard moves to the attacked vertex. The γ–θ conjecture asserts that $\gamma(G) = \gamma^{\infty}(G)$ implies $\gamma(G) = \theta(G)$. We prove the finite theorem that no graph on 13 vertices satisfies

$$\gamma(G) = \gamma^{\infty}(G) = 3 < \theta(G).$$

The proof fixes a maximum independent triple in an optimal eternal family and divides the search into full-response and no-full-response branches. The full-response branch is excluded by a 9,802-variable formula and an independently replayed RUP refutation. In the no-full branch, human one-guard lemmas force two distinct response types and pure physical representatives. A separate 1,222-variable certificate bounds the neutral vertices by three; a final 9,802-variable formula then covers all remaining signature multisets. Its deletion-free proof has 156,205 RUP additions and zero RAT lemmas. Independent checkers reconstruct the decisive formulas byte for byte, verify the symmetry and coverage arguments, replay the proofs, and check satisfiable ablations and sharp controls. This result does not exclude parameters four or five at order 13 and does not resolve the universal conjecture.

1 Introduction

Let G be a finite simple graph. The domination number $\gamma(G)$ is the minimum size of a set whose closed neighborhood is all of $V(G)$. The clique cover number $\theta(G)$ is the minimum number of parts in a partition of $V(G)$ into cliques; equivalently,

$$\theta(G) = \chi(\overline{G}). \tag{1.1}$$

In the standard one-guard eternal domination game, guards occupy a dominating set. After an attack at an unoccupied vertex, exactly one adjacent guard moves to the attacked vertex. The least number of guards that can answer every finite attack sequence is $\gamma^{\infty}(G)$.

Klostermeyer and MacGillivray raised the implication

$$\gamma(G) = \gamma^{\infty}(G) \quad \Longrightarrow \quad \gamma(G) = \theta(G) \tag{1.2}$$

[2]. Klostermeyer and Mynhardt later identified a gap in that argument and reopened the implication as an explicit question [3]. Subsequent literature refers to their conjectural formulation as the γ–θ conjecture [4]. MacGillivray, Mynhardt, and Virgile reported an exhaustive computation finding no counterexample through order 11 [4]. A separate companion paper gives a certificate-backed exclusion at order 12. The present paper advances one parameter slice at the next order.

Theorem 1.1 (order 13, parameter three)**.** *There is no finite simple graph G on 13 vertices such that*

$$\gamma(G) = \gamma^{\infty}(G) = 3 < \theta(G). \tag{1.3}$$

Theorem 1.1 is unconditional on lower-order enumeration: it excludes exactly the displayed $(n, k) = (13, 3)$ slice. In combination with the preceding order-12 frontier and the standard structural bounds used there, a counterexample of order 13, if one exists, has common parameter four or five. Neither statement proves the conjecture for all graphs.

The proof is both mathematical and certificate-backed. Section 2 fixes the exact game and derives the equality structure. Section 3 proves the response-list lemmas that cover the no-full branch. Sections 4–6 give the three finite certificates and their coverage arguments. Section 7 assembles the theorem, and Section 8 records exact hashes and reproduction instructions.

Remark 1.2 (status and priority)*.* This manuscript and its computational artifacts were produced in an independent, AI-assisted research campaign and have not been externally peer reviewed. The literature audit found no prior order-13, parameter-three exclusion, but no worldwide priority claim is made.

2 The exact model and equality structure

Definition 2.1. An *eternal dominating family of size k* is a nonempty family $\mathcal{D} \subseteq \binom{V(G)}{k}$ such that:

1. every $D \in \mathcal{D}$ dominates G; and
2. for every $D \in \mathcal{D}$ and every $r \in V(G) - D$, there is a guard $u \in D \cap N_G(r)$ for which

$$(D - \{u\}) \cup \{r\} \in \mathcal{D}. \tag{2.1}$$

The minimum such k is $\gamma^{\infty}(G)$.

Thus no occupied-vertex attacks are imposed, and a legal response changes exactly one guard position along exactly one edge. We do not use an all-guards-move, total, connected, eviction, or distance variant.

Proposition 2.2 (parameter chain)**.** *For every nonempty graph,*

$$\gamma(G) \leq i(G) \leq \alpha(G) \leq \gamma^{\infty}(G) \leq \theta(G). \tag{2.2}$$

Consequently, if $\gamma(G) = \gamma^{\infty}(G) = k$, then

$$\gamma(G) = i(G) = \alpha(G) = \gamma^{\infty}(G) = k. \tag{2.3}$$

Proof. Every maximal independent set dominates, and a maximum independent set is maximal. To see $\alpha \leq \gamma^{\infty}$, attack successively at unoccupied vertices of a maximum independent set I. No guard already on I can answer another attack in I, so every response increases the number of guards on I. Fewer than $|I|$ guards eventually leave an unoccupied vertex of I with no possible response.

Given a clique partition, place one guard in each part. An attack is answered by moving the guard within the attacked vertex's clique. This proves $\gamma^{\infty} \leq \theta$, and squeezing gives (2.3). □

The preceding attack argument also gives a fact used repeatedly below.

Lemma 2.3 (independent states are forced)**.** *Every independent k-set belongs to every eternal family of k-sets.*

Proof. Start from any family state and attack the unoccupied vertices of the independent k-set. Each response increases its occupancy by one, so closure eventually reaches the whole set. $\square$

We use two elementary eternal-domination facts and one standard small-parameter consequence in the complement.

Lemma 2.4 (induced-subgraph monotonicity)**.** *If J is a nonempty induced subgraph of G, then*

$$\gamma^{\infty}(J) \leq \gamma^{\infty}(G). \tag{2.4}$$

Proof. Fix an eternal family in G, let m be the maximum number of guards that any family state places in $V(J)$, and retain the states attaining m. An attack inside J cannot be answered by moving a guard from outside J, since that would produce a family state with $m+1$ guards in J. Hence every response stays inside J and preserves m. Projecting the retained states to J gives a closed family of m-sets. Every projected state dominates J: an unoccupied vertex has an adjacent responding guard in J, while an occupied vertex dominates itself. Also $m > 0$, since an attack in nonempty J would otherwise move a guard into J. Thus the projections form an eternal family in J. $\square$

Lemma 2.5 (odd antiholes)**.** *For every odd $\ell \geq 5$,*

$$\gamma^{\infty}(\overline{C_\ell}) = 3. \tag{2.5}$$

Proof. The upper bound follows from $\theta(\overline{C_\ell}) = \chi(C_\ell) = 3$. Label the underlying cycle cyclically by $0, \dots, \ell-1$. A pair fails to dominate $\overline{C_\ell}$ exactly when its two vertices have cyclic distance two.

Suppose an eternal two-family existed. Its independent pair $\{0,1\}$ belongs to the family by Lemma 2.3. If $\{0,d\}$ belongs to the family for odd $1 \leq d \leq \ell-6$, attack $d+2$. Moving the guard at 0, if legal, leaves the nondominating pair $\{d, d+2\}$. Closure therefore forces the other guard to move, and $\{0, d+2\}$ belongs to the family. Iteration reaches $\{0, \ell-4\}$. Attacking $\ell-2$ then leaves a cyclic-distance-two pair whichever guard moves. For $\ell = 5$, this last attack applies directly to the initial pair. $\square$

Lemma 2.6 (parameter two)**.** *If $\alpha(J) = \gamma^{\infty}(J) = 2$, then $\theta(J) = 2$.*

Proof. Suppose $\theta(J) > 2$, and put $K = \overline{J}$. Then $\omega(K) = 2 < \chi(K)$, so the Strong Perfect Graph Theorem [1] supplies an induced odd hole or odd antihole. An odd hole in K induces an odd antihole in J, contradicting Lemmas 2.5 and 2.4. An odd antihole in K induces an odd hole in J. At length at least seven this contradicts $\alpha(J) = 2$; at length five, self-complementarity and Lemmas 2.5 and 2.4 again contradict $\gamma^{\infty}(J) = 2$. Hence $\chi(K) = 2$. $\square$

Remark 2.7. Lemma 2.6 is used below only to turn an exact two-guard projection into a bipartite complement.

3 Response lists and the no-full decomposition

Assume for the remainder of Sections 3–6 that a graph and family satisfy

$$|V(G)| = 13, \qquad \gamma(G) = \alpha(G) = \gamma^{\infty}(G) = 3 < \theta(G). \tag{3.1}$$

Put $H = \overline{G}$, fix an eternal family $\mathcal{F}$ of triples, and fix a maximum independent state

$$S = \{a, b, c\} \in \mathcal{F} \tag{3.2}$$

using Lemma 2.3. For $x \notin S$, define its family-response list and anchor signature by

$$L(x) = \{u \in S : S - u + x \in \mathcal{F}\}, \qquad \sigma(x) = N_H(x) \cap S. \tag{3.3}$$

An attack at x from S proves $L(x) \neq \varnothing$. Moreover

$$L(x) \subseteq S - \sigma(x), \tag{3.4}$$

because $S - u + x$ must dominate the omitted anchor u, while the other two anchors do not see u. Put

$$Q = \{x \notin S : \sigma(x) = \varnothing\}, \qquad A = (V(G) - S) - Q. \tag{3.5}$$

The vertices in Q are G-complete to S and will be called *neutral*.

3.1 Frozen projections and response types

For $u \in S$, let

$$W_u = \{x \notin S : u \notin L(x)\}. \tag{3.6}$$

Lemma 3.1 (restoration)**.** *If $D \in \mathcal{F}$ and $u \in S - D$, then*

$$u \in \bigcup_{x \in D - S} L(x). \tag{3.7}$$

Proof. Starting afresh from D, attack every vertex of $(S - D) - \{u\}$. A guard already on S cannot answer another attack in S, because S is independent. These attacks therefore restore all missing anchors except u by moving guards from $D - S$. The resulting family state has the form $S - u + x$ for some $x \in D - S$, so $u \in L(x)$. □

Lemma 3.2 (frozen projection)**.** *The complement-induced graph*

$$H[(S - \{u\}) \cup W_u] \tag{3.8}$$

is bipartite.

Proof. Let $R = (S - \{u\}) \cup W_u$, and define

$$\mathcal{P}_u = \{A \in \tbinom{R}{2} : \{u\} \cup A \in \mathcal{F}\}.$$

This family is nonempty because $S - \{u\} \in \mathcal{P}_u$. Take $A \in \mathcal{P}_u$ and attack $r \in R - A$ from $\{u\} \cup A$. If u answered, the successor $A \cup \{r\}$ would omit u, while every one of its outside vertices belongs to W_u. Lemma 3.1 applied to that successor would force one of those outside vertices to have u in its response list, a contradiction. Hence a guard in A moves, and the resulting two-set remains in $\mathcal{P}_u$. The same response supplies an adjacent guard for every unoccupied vertex of R, so every member of $\mathcal{P}_u$ dominates $G[R]$. The projected states therefore form an eternal two-family in the subgraph of G induced by R.

The set $S - \{u\}$ is independent there, so that induced graph has $\alpha = \gamma^{\infty} = 2$. Lemma 2.6 gives clique cover number two; equivalently, (3.8) is bipartite. □

Lemma 3.3 (singleton safety)**.** *If $L(x) = L(y) = \{u\}$, then $xy \in E(G)$.*

Proof. If $xy \in E(H)$, attack y from $S-u+x$. The guard at x cannot move, so an anchor $v \in S-\{u\}$ moves to y, producing $S-\{u,v\}+\{x,y\}$. Attack the now-unoccupied u. No remaining anchor can answer. A response by x or y produces a direct swap at the reference state and puts v into $L(y)$ or $L(x)$, contrary to both lists being $\{u\}$. □

Say that *type u* occurs if some vertex has the exact two-list

$$L(x) = S - \{u\}. \tag{3.9}$$

Lemma 3.4 (at least two types)**.** *If every outside list has size at most two, then at least two distinct two-list types occur.*

Proof. Suppose at most one type occurs, and choose $c \in S$ so that the occurring type, if any, is type c. Every vertex outside $(S - \{c\}) \cup W_c$ has $c \in L(x)$. Under the no-full hypothesis it must have singleton list $\{c\}$, since any two-list containing c would have omitted color different from c. Therefore

$$V(H) = \big((S - \{c\}) \cup W_c\big) \,\dot\cup\, \big(\{c\} \cup \{x : L(x) = \{c\}\}\big). \tag{3.10}$$

The first induced part is bipartite by Lemma 3.2. The second is independent in H: response membership gives the edges cx in G, and Lemma 3.3 gives every edge between its outside vertices. Thus H is 3-colorable, contradicting $\theta(G) = \chi(H) > 3$. □

3.2 Physical representatives

Lemma 3.5 (two-response replication)**.** *Let $x \in Q$ and suppose $a, b \in L(x)$. There is a vertex $z \in A$ such that*

$$L(z) = \{a, b\}, \qquad \sigma(z) = \{c\}. \tag{3.11}$$

Proof. Because $\gamma(G) \geq 3$, the pair $\{c, x\}$ does not dominate. Choose $y \in N_H(c) \cap N_H(x)$. The retained states $\{b, c, x\}$ and $\{a, c, x\}$ both dominate y, forcing $ay, by \in E(G)$, hence $\sigma(y) = \{c\}$.

The pair $\{c, y\}$ also fails to dominate, so choose $z \in N_H(c) \cap N_H(y)$. If z missed a in G, domination of $\{a, c, x\}$ would force $xz \in E(G)$. Attacking z from $\{b, c, x\}$ would then leave no response: c cannot move; moving x leaves a state missing a; and moving b leaves a state missing y. The case in which z misses b is symmetric. Therefore $\sigma(z) = \{c\}$.

Attack z from each of $\{b, c, x\}$ and $\{a, c, x\}$. The guard at c cannot move, and moving the other anchor again leaves $\{c, x, z\}$, which misses y. Closure forces the guard at x to move in both attacks. Thus $xz \in E(G)$ and both $S - a + z, S - b + z$ belong to $\mathcal{F}$. Equation (3.4) excludes c from $L(z)$, proving (3.11). □

Corollary 3.6 (physical representative)**.** *Every occurring type u has a representative z_u satisfying*

$$L(z_u) = S - \{u\}, \qquad \sigma(z_u) = \{u\}. \tag{3.12}$$

Proof. Let t have list $S - \{u\}$. If $ut \in E(H)$, take $z_u = t$. Otherwise (3.4) shows that $t \in Q$, and Lemma 3.5, after relabeling the anchors, applies. □

Lemma 3.7 (pure-signature doubling)**.** *For every occurring type u, there are distinct vertices z_u, w_u such that*

$$\sigma(z_u) = \sigma(w_u) = \{u\}, \qquad z_u w_u \in E(H), \qquad L(z_u) = S - \{u\}. \tag{3.13}$$

Proof. Take z_u from Corollary 3.6. Since $\{u, z_u\}$ does not dominate, choose $w_u \in N_H(u) \cap N_H(z_u)$. It lies outside S and has u in its anchor signature. If another anchor r also lay in $\sigma(w_u)$, let h be the third anchor. The retained state $S - h + z_u = \{u, r, z_u\}$ would miss w_u, a contradiction. Thus $\sigma(w_u) = \{u\}$, and the chosen common-neighbor incidence gives the required H-edge. □

By Lemmas 3.4 and 3.7, after permuting the three anchors and relabeling outside vertices, every no-full candidate has

$$\begin{array}{c|c|l} \text{vertices} & \text{signature} & \qquad\text{additional condition} \\ \hline 3,4 & \{0\} & 34 \in E(H), \quad L(3) = \{1,2\}, \\ 5,6 & \{2\} & 56 \in E(H), \quad L(5) = \{0,1\}. \end{array} \tag{3.14}$$

This is the structural input to the final certificates.

4 The full-response branch

A vertex $x \notin S$ is a *full target* if $L(x) = S$.

Theorem 4.1 (certified full-response exclusion)**.** *No object satisfying (3.1) has a full target at S.*

Certificate and coverage proof. Relabel $S = 012$ and a specified full target as $x = 3$. Edge variables encode $H = \overline{G}$. The formula has four variable families:

$$\begin{array}{l|r} H\text{-edges} & 78 \\ \text{pair common-neighbor selectors} & 858 \\ \text{retained triples} & 286 \\ \text{one-guard response selectors} & 8,580 \\ \hline \text{total} & 9,802. \end{array} \tag{4.1}$$

Its 85,409 clauses enforce: no K_4 in H; an outside common H-neighbor for every pair; domination of every retained triple; exact one-guard closure for each unoccupied attack; the independent retained anchor triangle and three full successors; and non-3-colorability of H.

The coloring block is complete. A proper 3-coloring gives three distinct colors to the anchor triangle; permute their names to $0, 1, 2$. The formula blocks all $3^{10} = 59,049$ assignments on the remaining vertices by requiring a monochromatic H-edge in each assignment.

Vertices $4, \ldots, 12$ are otherwise interchangeable. Sort their four-bit H-adjacency signatures to $0, 1, 2, 3$. The eight adjacent comparators use $8\binom{16}{2} = 960$ clauses. Every nonsorter clause family is covariant under the full S_9 action, including the complete coloring bank, so every unrestricted object has a sorted representative. An independent truth table checked all 2,048 local signature pairs.

A clean-room generator reconstructed the 4,808,845-byte DIMACS file byte for byte. Its SHA-256 is

`d5a2f17ad6e61cb7ca5cb9d2930b6a0738fec32ee1d9956207dc67bb297dcb13`.

The 19,874,489-byte DRAT proof has SHA-256

`653b01e904b97c01bfa25fbbea29fbadee603918dbaff0ea41b7ad09460fb910`.

Independent replay reported `s VERIFIED` and zero RAT lemmas; a separately retained reduced proof also replayed. Since any hypothetical object maps to a model of the reconstructed formula, its certified unsatisfiability proves the theorem. □

Removing the 59,049 coloring clauses makes the formula satisfiable. The decoded graph has graph6 record `LF\|ul\XzVsaqJ` in the source archive, parameters $(\gamma, \alpha, \gamma^\infty, \theta) = (3, 3, 3, 3)$, and all 157 dominating triples in its greatest eternal family. This positive control shows that the full-response and equality clauses are not intrinsically inconsistent.

5 Four neutral vertices are impossible

Theorem 5.1 (four-neutral/two-port exclusion)**.** *On 13 vertices, there is no graph with $\gamma(G) \geq 3$, an independent retained triple $S = \{h, i, j\}$, four distinct neutral vertices, and two further distinct vertices x, y satisfying*

$$\{h, j\} \subseteq L(x), \qquad \{h, i\} \subseteq L(y), \tag{5.1}$$

that admits an eternal family of triples.

Certificate and scope proof. The formula deliberately does not impose a clique-cover gap, connectedness, no-full lists, negative response memberships, signatures of x, y, or greatest-family status. It uses 78 edge variables, 286 family variables, and 858 common-neighbor selectors. Exact one-guard closure is expanded directly into eight clauses per retained-state/attack pair. The total is

$$1,222 \text{ variables}, \qquad 24,694 \text{ unique, nontautological clauses.} \tag{5.2}$$

An independent reconstructor produced the exact 447,906-byte instance with SHA-256

`3d1a1379eb2a90ffd399e5a830b1a81881ed527c6e9db06574a390085cb5c1e0`.

Its deletion-free proof contains 78,697 additions, ends in the empty clause, and has SHA-256

`c4f1989ac80474a86b75ba939e494bde5928b2727fd61297eb695f3937222eee`.

Strict forward replay verified every addition by reverse unit propagation, reported zero RAT lemmas, and a fresh DRAT-to-LRAT conversion passed an independent LRAT checker.

There is no symmetry assumption in this formula. Any object in the theorem can be relabeled to its named anchors, ports, and four neutral vertices, so the certificate covers the stated universe directly. □

Corollary 5.2. *Every no-full candidate satisfying (3.1) has*

$$|Q| \leq 3, \qquad |A| \geq 7. \tag{5.5}$$

Proof. Choose the two types and their distinct representatives in (3.14). Their two response pairs overlap in one anchor. If four neutral vertices existed, Theorem 5.1 would apply after relabeling. There are ten vertices outside S, so the bound on A follows. □

The neutral bound is sharp with respect to the certificate hypotheses. A checked 13-vertex control has graph6 record `LDZZa^g|fkw[iH`,

$$\gamma = i = \alpha = \gamma^\infty = \theta = 3, \tag{5.6}$$

and a greatest eternal family of 139 triples. It realizes three neutral vertices and both positive port pairs.

6 The residual no-full certificate

Theorem 6.1 (certified residual exclusion)**.** *No no-full object satisfying (3.1) exists.*

Formula coverage and certificate. Use the normalization (3.14). Label the six interchangeable remaining vertices $7, \ldots, 12$. Sort their three-bit anchor signatures in nondecreasing order. Since the four named vertices are nonneutral, Corollary 5.2 leaves at most three neutral vertices among the six residual labels. The fourth residual vertex, label 10, therefore has nonzero signature.

This normalization covers every candidate. There are six ordered choices of two distinct omitted colors, all transported to the named pair by one of the six anchor permutations. For the residual vertices, sorting covers all

$$\binom{8+6-1}{6} = 1,716 \tag{6.1}$$

signature multisets, including ties. The label-10 condition is equivalent to the at-most-three-neutral bound in this sorted order.

The exact formula again has 9,802 variables. Its 84,614 clauses have the following independently reconstructed census:

anchor H-triangle	3	anchor state retained	1
no H-K_4	715	pair witness choice/edges	$78 + 1,716$
selected-state domination	$2,860$	selected-state response	$2,860$
move uses a G-edge	$8,580$	move reaches a retained state	$8,580$
anchored non-3-colorability	$59,049$	no-full target clauses	10
named lists, signatures, edges	$6 + 12 + 2$	residual signature sorter	140
at most three neutral residuals	1	family nonempty	$1.$

(6.2)

Response variables name one occupied guard and one unoccupied attack. Their implications require the corresponding G-edge and the unique one-guard successor. Thus multiple true response variables represent alternative legal moves, never simultaneous guard motion.

The clean-room reconstruction is byte-identical to the 4,784,714-byte DIMACS file. Its SHA-256 is

`76ff2768c7afd95ee535f8684515b0b15319b1f5ca69085447a1f7eba66393e1`.

The retained raw DRAT trace was replayed both with and without honoring deletion lines. Removing those lines gives a proof containing exactly 156,205 additions, 8,878,465 bytes, and SHA-256

`c985ce0a602a91a0d323594e3aeecf210fa5131027ef4b6c9b6e4d4b628f1848`.

Strict forward, warning-fatal replay reported `s VERIFIED` and zero RAT lemmas. The clean-room audit separately checked variable ranges, clause multiplicities, absence of duplicates and tautologies, all six anchor normalizations, all 1,716 signature multisets, and the exact sorter truth table. Hence no normalized candidate survives. ☐

Removing the 59,049 clique-cover clauses makes the residual formula satisfiable, and the retained assignment satisfies every remaining clause. Thus the certificate does not merely prove that the equality and structural normalization are inconsistent.

7 Assembly and exact scope

Proof of Theorem 1.1. Suppose (1.3) holds. Proposition 2.2 gives

$$\gamma = i = \alpha = \gamma^\infty = 3.$$

Choose an optimal eternal family and a maximum independent triple S. Every outside response list is nonempty. If some list is full, Theorem 4.1 is a contradiction. Otherwise every list has size at most two, and Theorem 6.1 is a contradiction. The two branches exhaust all possibilities. ☐

Corollary 7.1. *Relative to the published through-order-11 result and the separately certified order-12 theorem, a counterexample of order 13, if one exists, has common parameter four or five.*

Proof. The parameter chain and Lemma 2.6 give common parameter at least three; Theorem 1.1 excludes three. The no-simplicial-vertex theorem in the order-12 companion paper gives minimum degree at least two for a minimum counterexample. The standard domination bound for connected minimum-degree-two graphs [5] then gives $\gamma(G) \leq \lfloor 2|V(G)|/5 \rfloor = 5$ at order 13. Thus only four and five remain. ☐

Remark 7.2 (what has not been proved). Finite verification does not resolve (1.2). In particular, this paper does not exclude order-13 parameters four or five, any parameter at order 14, or parameter three at arbitrary order. It supplies no counterexample and makes no claim about the all-guards-move model.

8 Reproduction and independent checks

The source archive is

`github.com/AlecKriebel/Math`,

with frozen release tag

`gamma-theta-order13-k3-v1.0.0`.

From the campaign directory, the compact aggregate replay is:

```
python3 -I -B -W error repro/c097/replay.py
```

It binds the decisive hashes, replays the four-neutral and residual proofs, reconstructs both formulas, verifies the normalization counts, checks the sharp three-neutral equality control, and checks the satisfiable theta-gap ablation.

The branch-specific independent checkers are:

```
python3 -I -B -W error reviews/order13_full_target_hostile/checker.py
python3 -I -B -W error reviews/tight_micro_hostile_review/checker.py
python3 -I -B -W error reviews/order13_no_full_a7_hostile/checker.py
```

The discovery generators do not serve as their own decisive validators. The review programs use separately written allocation and clause-generation logic, inspect the graph/complement direction, truth-table the closure and sorter gadgets, and replay pinned proof bytes with independently hashed checkers. Satisfiable controls are checked by direct clause evaluation and, where graph parameters are reported, by a separate ordinary-set evaluator.

Branch	variables	clauses	decisive proof type
full response	9,802	85,409	RUP-only DRAT, plus reduced replay
four neutral	1,222	24,694	78,697 addition-only RUP steps
residual no-full	9,802	84,614	156,205 addition-only RUP steps

Table 1: The three finite certificates used in the proof.

AI-assistance and verification disclosure

The exploratory mathematics, programs, certificate design, adversarial audits, literature review, manuscript, and publication materials were developed with heavy assistance from ChatGPT 5.6 Sol under Alec Kriebel's direction. Alec Kriebel is the human author and project lead. No outside individual was contacted for this project, and no external expert has reviewed the result. No claim was accepted on model output alone: the finite statements are bound to exact formulas, proof objects, independent reconstructors, and direct controls. Those checks are evidence about the encoded theorem; they are not peer review.

Acknowledgment of prior work

The through-order-11 computation is due to Gary MacGillivray, C. M. Mynhardt, and V. Virgile. The original question is due to William F. Klostermeyer and Gary MacGillivray, and its later formulation as the γ–θ conjecture is due to William F. Klostermeyer and C. M. Mynhardt.

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