

Reversible equilibrium continua without a common factor

Version 2 specialist audit statement | exact three-species construction and rate family

Problem / answer

Problem. Can a finite weakly reversible mass-action system have a positive-dimensional continuum of positive equilibria in one compatibility class while $\gcd(F_1, F_2, F_3)=1$? **Answer: yes.** The explicit system below is reversible, has one linkage class, three species, ten complexes, ten reversible pairs, positive integer rates, and $S=R^3$. Its unique positive class is the positive orthant.

Main explicit theorem

Complexes: 0, Z, 3Z, Y+Z, 3Y, X+Z, X+Y, X+Y+Z, 2X+Y, 3X. Reversible edges: 01, 04, 06, 17, 24, 27, 29, 34, 59, 89. Directed order is forward then reverse in that edge order.

```
1160, 10296, 976, 23, 560, 5977, 1800, 25, 1629, 1237,
1, 9152, 653, 1214, 5368, 1, 5368, 70, 6039, 915
```

The equilibrium ellipse is $L=z-x-y+1=0$ and $Q=7x^2-2xy-16x+7y^2-16y+16=0$. It is compact and entirely positive. A rational parametrization is $x=(t^2+3)/(2D)$, $y=(3t^2+1)/(2D)$, $z=(t^2+t+1)/D$, with $D=t^2-t+1$.

The reconstructed field is:
 $F_1=-4697x^3+6039x^2y-9177xyz-5977xy+10736xz+1960z^3+1800z+560$;
 $F_2=915x^3-6039x^2y-9177xyz-5977xy-3782y^3+10736yz+4888z^3+1800z+3488$;
 $F_3=3712x^3+18304xyz-5368xz+3712y^3-5368yz-6848z^3-10296z+1160$.

Exactly: F_i belongs to (L, Q) , $\gcd(F_1, F_2, F_3)=1$ over Q , R , and C , and the steady ideal is radical. Its components are the conic prime and a disjoint degree-15 maximal ideal (15 reduced isolated points over an algebraic closure).

Complete fixed-support family theorem

On the same 20 directed edges, all conic-preserving rate vectors form a four-dimensional rational linear space. Free coordinates are $(a, b, c, d)=(k29, k43, k95, k98)$. The other 16 rates are explicit rational linear forms certified by a canonical 21×20 remainder matrix of rank 16. All 20 rates are positive exactly when $a, b, c, d > 0$, $b < c$, and $192a + 221c < 154d$.

A nonempty Zariski-open subset of this positive family is geometrically coprime. The displayed system is $(a, b, c, d)=(653, 1, 70, 915)$. Under primitive positive-integral normalization and within this fixed support/family, it simultaneously minimizes the maximum rate (10296) and rate sum (52464). No global support-minimality claim is made.

Reaction table and exact replay

Complete table: manuscript_v2_draft/rates.csv. Human-readable table: MANUSCRIPT_V2.md, Appendix A. Family formulas and rank certificate: family/README.md and family/remainder_matrix.csv.

```
.venv/bin/python weakly_reversible_continuum_no_common_factor/
manuscript_v2_draft/verify_v2_claims.py
```

Likely failure points already tested

- edge/rate ordering, reverse edges, graph connectivity, and linkage count;
- positive rates, exact rank three, and the full-orthant compatibility class;
- conic primeness, compact positivity, parametrization, and exact vanishing;
- affine and homogenized gcd, including scalar extension to R and C ;
- family exhaustivity, positivity sufficiency, and origin/degree-drop cases;
- radical decomposition, residual separability, and disjointness;
- bounded integer optimum without floating point; and
- strict separation of frozen-rate stability claims from the clean system.

Metadata and scope

Repository-wide Zenodo concept DOI: **10.5281/zenodo.21753404**. It groups unrelated releases from the AlecKriebel/Math monorepo and is not a paper-specific all-versions DOI. Cite this paper using its Version 2 version-specific DOI: **pending the final GitHub release**. This packet does not publish, deposit, or contact anyone. A DOI is not a correctness certificate.

Not claimed: global minimality in complexes/reactions/deficiency; persistence outside the four-dimensional family; or universal priority. See audit_packet/AUDIT_CHECKLIST.md for the adversarial test map.