Simultaneous amplification beyond fitness three halves

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Abstract

We study fixation from a uniformly random single mutant on finite connected loopless undirected weighted graphs under both Birth–death (Bd) and death–Birth (dB) Moran updating. Let R_{hyb} be the unique root in $(3/2, 151/100)$ of

$$R^6 - 8R^5 + 22R^4 - 30R^3 + 21R^2 - 6R + 1.$$

We construct one fitness-independent graph family which, for every fixed $1 < r < R_{\mathrm{hyb}}$, strictly amplifies fixation under both rules for all sufficiently large populations. Numerically, $R_{\mathrm{hyb}} = 1.5028569127905696\ldots$, and hence $R_{\mathrm{sim}} \geq R_{\mathrm{hyb}} > 3/2$.

The graph combines a large clique, a dilute collection of hub pendants, and a dilute collection of internally heavy two-vertex satellites coupled to the clique through positive weak edges. We first derive the exact finite weak-cut trace. We then control establishment and the entire path to fixation, obtaining the normalized corrections

$$B(r) = \frac{2(\sigma-1)}{1+\sigma(r^2-1)} + \frac{\lambda}{r-1}, \qquad D(r) = \frac{2\{r(2-r)-\sigma\}}{\sigma+2r(r-1)} - \lambda.$$

Optimizing the interval on which both are positive gives the displayed sextic. Independent labelled-event and symbolic verifiers audit the lumping, trace coefficients, root isolation, and tangency.

This disproves the proposed exact threshold 3/2. We also exactly refute every graph-independent convex affine endpoint separator. The unrestricted exact value of R_{sim} remains open.

Scope. This paper proves the lower bound $R_{\mathrm{sim}} \geq R_{\mathrm{hyb}} = 1.5028569127905696\ldots > 3/2$ and the exact threshold of the displayed two-mechanism dilute hybrid. It does *not* prove a universal upper bound or determine the exact unrestricted value of R_{sim}.

Author summary

The two standard update orders in evolutionary graph theory often reward opposite structural features. We combine two small effects at a vanishing population proportion. Internally heavy pairs improve death–Birth fixation but slightly reduce Birth–death fixation. Hub pendants have the opposite effect. Their proportions can be balanced so that both net effects are positive, even at fitness 3/2 and on a short interval above it. Exact optimization produces an algebraic endpoint near 1.5028569. The weak links are chosen uniformly over a growing fitness interval, so the graph sequence is fixed before fitness is quantified.

1 Introduction

A population structure is a simultaneous amplifier at fitness r when it raises uniformly initialized fixation probability, relative to the complete graph of the same order, under both Bd and dB updating. The asymptotic question has the quantifier order

$$\exists\{G_t\}\ \forall r \in (1,R)\ \exists t_0(r)\ \forall t \geq t_0(r).$$

The graphs and weights may depend on t, but not on r.

Svoboda, Joshi, Tkadlec, and Chatterjee constructed simultaneous amplifiers on the interval $1 < r < 1.2$ [1]. A separate center–triangle construction subsequently established $R_{\mathrm{sim}} \geq 3/2$, while that particular family suppresses at the endpoint [2]. The latter fact suggested, but did not prove, that $3/2$ might be the universal phase boundary.

The present construction crosses that boundary. The mechanism is a dilute linear combination of two rigorously traced modules. Its exact optimization also identifies the best threshold within this two-mechanism leading regime. Two endpoint proof programs are thereby closed: universal impossibility at $3/2$ is false, and an exact order-22 graph rules out every fixed convex affine separator of the two normalized endpoint probabilities.

2 Model, baselines, and threshold

Let $G = (V, w)$ be finite, connected, loopless, undirected, and weighted:

$$w_{uv} = w_{vu} \geq 0, \qquad w_{uu} = 0, \qquad d_u := \sum_v w_{uv} > 0.$$

A state is a mutant set $S \subseteq V$. Mutants have fitness $r > 1$ and residents fitness one. Under Bd, a parent is selected proportional to fitness and then a replacement target proportional to edge weight. Under dB, a uniformly chosen target dies and then a parent is selected proportional to fitness times edge weight. Thus, with $F(S) = |V| + (r-1)|S|$,

$$P_{\mathrm{Bd}}(S, S \cup \{v\}) = \frac{r}{F(S)} \sum_{u \in S} \frac{w_{uv}}{d_u}, \qquad v \notin S, \tag{1}$$

$$P_{\mathrm{Bd}}(S, S \setminus \{v\}) = \frac{1}{F(S)} \sum_{u \notin S} \frac{w_{uv}}{d_u}, \qquad v \in S, \tag{2}$$

$$P_{\mathrm{dB}}(S, S \cup \{v\}) = \frac{1}{|V|} \frac{r \sum_{u \in S} w_{uv}}{r \sum_{u \in S} w_{uv} + \sum_{u \notin S} w_{uv}}, \qquad v \notin S, \tag{3}$$

$$P_{\mathrm{dB}}(S, S \setminus \{v\}) = \frac{1}{|V|} \frac{\sum_{u \notin S} w_{uv}}{r \sum_{u \in S} w_{uv} + \sum_{u \notin S} w_{uv}}, \qquad v \in S. \tag{4}$$

The fixation function h_U is the unique harmonic solution with boundary values zero at $\varnothing$ and one at V. We use

$$\rho_U(G, r) = \frac{1}{|V|} \sum_{v \in V} h_U(\{v\}), \qquad U \in \{\mathrm{Bd}, \mathrm{dB}\}.$$

Direct solution of the one-dimensional complete-graph count chains gives

$$\rho_{\mathrm{Bd}}(K_n, r) = \frac{1 - r^{-1}}{1 - r^{-n}}, \tag{5}$$

$$\rho_{\mathrm{dB}}(K_n, r) = \frac{n-1}{n} \frac{1 - r^{-1}}{1 - r^{-(n-1)}}. \tag{6}$$

Both converge, for fixed $r > 1$, to

$$p(r) := 1 - \frac{1}{r}.$$

Definition 1. R_{sim} is the supremum of all $R > 1$ for which one fitness-independent family $\{G_t\}$, with $|V(G_t)| \to \infty$, satisfies

$$\rho_{\mathrm{Bd}}(G_t, r) > \rho_{\mathrm{Bd}}(K_{|V(G_t)|}, r), \qquad \rho_{\mathrm{dB}}(G_t, r) > \rho_{\mathrm{dB}}(K_{|V(G_t)|}, r)$$

for every fixed $r \in (1, R)$ and every sufficiently large t.

3 The dilute pair–pendant hybrid

Let

$$P(R) = R^6 - 8R^5 + 22R^4 - 30R^3 + 21R^2 - 6R + 1$$

and let R_{hyb} be its unique root in $(3/2, 151/100)$. Define

$$\sigma_* = \frac{-R_{\mathrm{hyb}}^3 + 4R_{\mathrm{hyb}}^2 - 3R_{\mathrm{hyb}} - 1}{2(R_{\mathrm{hyb}} - 1)}, \tag{7}$$

$$\lambda_* = \frac{2(R_{\mathrm{hyb}} - 1)(1 - \sigma_*)}{1 + \sigma_*(R_{\mathrm{hyb}}^2 - 1)}. \tag{8}$$

Exact root isolation gives

$$R_{\mathrm{hyb}} = 1.5028569127905696\ldots, \quad \sigma_* = 0.1306772822870484\ldots, \quad \lambda_* = 0.7508064830318805\ldots.$$

For $t \geq 2$, put

$$q_t = t, \qquad m_t = \lfloor \lambda_* t \rfloor, \qquad C_t = t^4, \qquad n_t = C_t + m_t + 2q_t.$$

The graph contains:

1. a unit-weight clique on C_t vertices, one designated as the hub;
2. m_t unit-weight leaves adjacent only to the hub;
3. q_t disjoint two-vertex satellites, each with internal edge weight $W_t = C_t/\sigma_*$;
4. an edge of common weight $\varepsilon_t > 0$ from every satellite vertex to every clique vertex.

There are no other edges. In particular, there are no edges between distinct satellites.

To define the positive coupling without reference to a subsequently chosen fitness, let

$$I_t = [1 + 1/t, R_{\mathrm{hyb}} - 1/t]$$

when this interval is nonempty. Let $\rho^0_{U,t}$ denote the finite separated trace defined in Section 4. Choose e_t to be the least positive integer such that, for $\varepsilon_t = 2^{-e_t}$,

$$\sup_{r \in I_t} \frac{n_t}{q_t} \left| \frac{\rho_U(G_t(\varepsilon_t), r)}{\rho_U(K_{n_t}, r)} - \frac{\rho^0_{U,t}(r)}{\rho_U(K_{n_t}, r)} \right| \leq \frac{1}{t} \quad (U = \mathrm{Bd}, \mathrm{dB}).$$

For the finitely many empty intervals, take $e_t = 1$. Finite Schur-complement continuity proves existence. At fixed t, e, all entries are algebraic rational functions of r, denominators are positive, and the two uniform inequalities are decidable by exact real-algebraic arithmetic. Thus this is an effective graph definition, and every weight is fixed before r is quantified.

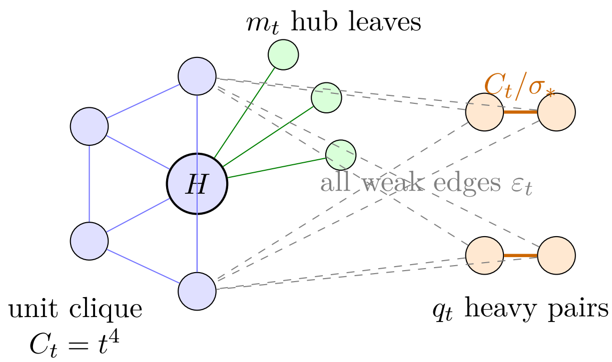

Figure 1: Schematic of the dilute hybrid. Dashed lines represent all satellite–clique edges, not only those drawn.

Theorem 2 (Main theorem)**.** *For every fixed* $1 < r < R_{\mathrm{hyb}}$*, the family above satisfies, for every sufficiently large* t*,*

$$\rho_{\mathrm{Bd}}(G_t, r) > \rho_{\mathrm{Bd}}(K_{n_t}, r), \qquad \rho_{\mathrm{dB}}(G_t, r) > \rho_{\mathrm{dB}}(K_{n_t}, r).$$

Consequently

$$\boxed{R_{\mathrm{sim}} \geq R_{\mathrm{hyb}} = 1.5028569127905696\ldots > 3/2.}$$

4 Exact lumping and the finite weak-cut trace

A configuration has orbit label

$$(h, i, u, v, \ell),$$

where h is the hub type, i the number of mutant ordinary clique vertices, u the number of mixed pairs, v the number of all-mutant pairs, and ℓ the mutant-leaf count. The group

$$S_{C_t-1} \times (S_2 \wr S_{q_t}) \times S_{m_t}$$

is transitive on every fibre and commutes with both update kernels. Therefore the orbit partition is a strong lumping: the total probability from a labelled state to any target fibre depends only on its five coordinates.

We first fix finite C, m, q and send $\varepsilon \downarrow 0$. At $\varepsilon = 0$ the closed fast classes are precisely the states in which the center module and every pair are internally homogeneous. Write a macro state as

$$(h, k) \in \{0, 1\} \times \{0, \ldots, q\},$$

where h is the center type and k the number of mutant pairs. Partition the finite first-step matrix into mixed-module states F and homogeneous states M. The $\varepsilon = 0$ block on F is invertible because every finite connected module absorbs internally. Its Schur complement on M, after removing the common first power of ε, converges to the introduction rate multiplied by the affected module's exact local fixation probability. This proves the trace from the finite absorbing system, without an independent-genealogy assumption.

Let a_H^U, a_P^U be the uniformly initialized isolated fixation probabilities of the center and one pair, and let P_U^H, P_U^P be macro fixation from $(1, 0)$ and $(0, 1)$. Then the finite separated limit is

$$\rho_U^0(C, m, q; r) = \frac{C + m}{N} a_H^U(r) P_U^H + \frac{2q}{N} a_P^U(r) P_U^P, \qquad N = C + m + 2q. \tag{9}$$

The convergence is uniform on every compact fitness interval because all finite denominators remain positive there.

5 Quantitative center-module estimates

Let $H_{C,m}$ be a unit clique K_C with m unit hub pendants and put $p = 1 - 1/r$. Along $C = t^4$, $m \asymp t$, the following estimates hold uniformly when r ranges over a compact subset of $(1, \infty)$:

$$u^U_{\mathrm{core}}(r) = p + o(q/C), \qquad U = \mathrm{Bd}, \mathrm{dB}, \tag{10}$$

$$u^{\mathrm{Bd}}_{\mathrm{leaf}}(r) = 1 - o(1), \tag{11}$$

$$u^{\mathrm{dB}}_{\mathrm{leaf}}(r) = O(C^{-1}). \tag{12}$$

Here u^U_{core} averages the C clique starting vertices. Consequently

$$a^{\mathrm{Bd}}_H(r) = \frac{Cp + m}{C + m} + o(q/C), \tag{13}$$

$$a^{\mathrm{dB}}_H(r) = \frac{Cp}{C + m} + o(q/C). \tag{14}$$

We give the quantitative argument because an $o(1)$ establishment approximation would be too weak: the final gain is only of order q/C. Write a center state as (h, i, ℓ), put $c = C - 1$, $d = c + m$, and suppress a common positive statewise time factor. The six Bd changing rates are

change	rate
$i \to i+1$	$r(c-i)\{h/d + i/c\}$
$i \to i-1$	$i\{(1-h)/d + (c-i)/c\}$
$0 \to 1$ at h	$r(i/c + \ell)$
$1 \to 0$ at h	$(c-i)/c + m - \ell$
$\ell \to \ell+1$	$rh(m-\ell)/d$
$\ell \to \ell-1$	$(1-h)\ell/d.$

After multiplication by $C + m$, the six dB rates are

$i \to i+1$	$(c-i)\dfrac{r(h+i)}{c+(r-1)(h+i)}$
$i \to i-1$	$i\dfrac{c-i+1-h}{c+(r-1)(i-1+h)}$
$0 \to 1$ at h	$\dfrac{r(i+\ell)}{d+(r-1)(i+\ell)}$
$1 \to 0$ at h	$\dfrac{d-i-\ell}{d+(r-1)(i+\ell)}$
$\ell \to \ell+1$	$h(m-\ell)$
$\ell \to \ell-1$	$(1-h)\ell.$

These tables follow by summing the labelled parent–target and death–parent events and are the only estimates used below.

Start from one ordinary clique mutant, with resident hub and no mutant leaves. Stop i on hitting 0 or $K = A \log C$. Before the first hub change, the exact embedded up/down odds are

$$R_{\mathrm{Bd}}(i) = r\,\frac{c-i}{c-i+c/(c+m)}, \tag{15}$$

$$R_{\mathrm{dB}}(i) = r\,\frac{c-i}{c-i+1}\,\frac{c+(r-1)(i-1)}{c+(r-1)i}. \tag{16}$$

Both equal $r\{1+O(K/C)\}$. At $i \le K$, the hub-change hazard divided by the ordinary-change hazard is $O(K/C)$. A biased walk started at one makes $O(K)$ expected ordinary changes before hitting $\{0,K\}$, so charging the first hub change as an error event costs $O(K^2/C)$.

The product-odds gambler's-ruin formula therefore gives

$$\mathbb{P}_1(T_K < T_0) = p + O(K^2/C) + O(r^{-K}).$$

For every hub and leaf state, the ordinary-count odds remain bounded below by a constant $r_0 > 1$ from K to a fixed positive core density. Indeed, the two tables give the uniform lower bounds

$$\frac{q^+_{\mathrm{Bd}}}{q^-_{\mathrm{Bd}}} \ge \frac{r}{1+c/\{d(c-i)\}}, \qquad \frac{q^+_{\mathrm{dB}}}{q^-_{\mathrm{dB}}} = r\,\frac{h+i}{i}\,\frac{c-i}{c-i+1-h}\,\frac{c+(r-1)(i-1+h)}{c+(r-1)(i+h)}.$$

Hence return to zero costs $O(r_0^{-K})$. In the upper strip the resident deficit is dominated by a subcritical linear birth–death chain. Recurrent mutant-hub blocks then fill all leaves: a dB leaf copies the hub at its next death, while the Bd excursion up/down ratio, for resident deficit $R = c-i$, is bounded below by

$$\frac{r^2(i/c+\ell)(m-\ell)}{(R/c+m-\ell)\ell} \ge \frac{r^2}{1+2\delta} > 1$$

once $R \le 2\delta c$, with δ chosen sufficiently small. Repeating $B \log C$ cleanup blocks makes failure $O(C^{-B})$, whereas a resident core lineage reaching positive density during these polynomially many blocks costs $\exp\{-\Omega(C)\}$.

Choose A and then B uniformly on the fitness compact so that all of

$$O(K^2/C) + O(r^{-K}) + O(r_0^{-K}) + O(C^{-B}) + \exp\{-\Omega(C)\}$$

are $o(q/C) = o(C^{-3/4})$. The hub starting vertex has weight $C^{-1} = o(q/C)$. This proves (10) and controls the route from establishment to fixation.

For a Bd mutant leaf, hub activation has order-one rate and leaf loss has rate $O(C^{-1})$. During an activated excursion, whose return rate is $m + O(1)$, a successful ordinary-core mark occurs with probability $\Theta(1/m)$. Resolving successive excursions gives loss-before-mark probability $O(m/C) + o(1)$; failed finite core families have a geometrically summable conditioned progeny tail. This proves (11). Under dB, the leaf dies at rate one after the common death-clock time change, whereas hub activation is at most r/C, proving (12).

At reciprocal fitness, the same stopped comparison is uniformly subcritical. Reaching a positive core density, necessary for local fixation, has probability $\exp\{-\Omega_r(C)\}$.

6 Pair gates and the complete macro path

For the center, let

$$I_H = \sum_{x \text{ in clique}} \frac{1}{d_x} = 1 + \frac{1}{C-1+m}, \qquad J_H(r) = \sum_{x \text{ in clique}} \frac{h_H(\{x\};r)}{d_x}.$$

For a pair with internal weight $W = C/\sigma$, put

$$I_P = \frac{2}{W} = \frac{2\sigma}{C}, \qquad J_P(r) = \frac{1}{W} = \frac{\sigma}{C}.$$

The second identity uses the exact dB singleton value $1/2$ on K_2. The center estimates imply

$$I_H = 1 + o(1), \qquad J_H(r) = p + o(1),$$

while $J_H(1/r)$ and the reciprocal core fixation probability are exponentially small.

For one specified pair, let A, D, B, C' denote, respectively, mutant-pair invasion of a resident center, resident-center recovery of that pair, mutant-center invasion of a resident pair, and resident-pair recovery of the center. Summing the parent–target or death–parent events gives, after a common time change,

	A	D	B	C'
Bd	$CrI_P a_{\text{core}}(r)$	$2I_H a_P(1/r)$	$2rI_H a_P(r)$	$CI_P a_{\text{core}}(1/r)$
dB	$2rJ_H(r)$	$(C/r)J_P(1/r)$	$CrJ_P(r)$	$(2/r)J_H(1/r).$

The isolated pair values are

$$a_P^{\text{Bd}}(r) = \frac{r}{r+1}, \quad a_P^{\text{Bd}}(1/r) = \frac{1}{r+1}, \quad a_P^{\text{dB}}(r) = a_P^{\text{dB}}(1/r) = \frac{1}{2}.$$

On macro states the only nonzero rates are

$$\begin{array}{ll} (0,k) \to (1,k) : kA, & (0,k) \to (0,k-1) : kD, \\ (1,k) \to (1,k+1) : (q-k)B, & (1,k) \to (0,k) : (q-k)C'. \end{array}$$

Since $qC'/B \to 0$, a mutant center converts all resident pairs with probability $1 - o(1)$. From one mutant pair, the successful gate odds tend to

$$Z_B = \sigma(r^2 - 1), \qquad Z_D = \frac{2r(r-1)}{\sigma}.$$

Therefore

$$P_U^H \to 1, \qquad P_{\text{Bd}}^P \to \frac{Z_B}{1+Z_B}, \qquad P_{\text{dB}}^P \to \frac{Z_D}{1+Z_D}.$$

This exact macro chain controls the post-establishment route; no branching survival probability is substituted for global fixation.

7 The simultaneous correction and its optimization

Put $\eta = q/C$. Substitution of (13)–(14) and the pair gates into (9), together with the $O(C^{-1}) = o(\eta)$ complete baseline correction, gives

$$\frac{\rho_{\text{Bd}}(G_t, r)}{\rho_{\text{Bd}}(K_{n_t}, r)} = 1 + \eta \left[\frac{2(\sigma-1)}{1+\sigma(r^2-1)} + \frac{\lambda}{r-1} \right] + o_r(\eta), \tag{17}$$

$$\frac{\rho_{\text{dB}}(G_t, r)}{\rho_{\text{dB}}(K_{n_t}, r)} = 1 + \eta \left[\frac{2\{r(2-r)-\sigma\}}{\sigma+2r(r-1)} - \lambda \right] + o_r(\eta). \tag{18}$$

The pair parts follow by the exact simplifications

$$2\left\{ \frac{[r/(r+1)]Z_B/(1+Z_B)}{p} - 1 \right\} = \frac{2(\sigma-1)}{1+\sigma(r^2-1)}, \tag{19}$$

$$2\left\{ \frac{[1/2]Z_D/(1+Z_D)}{p} - 1 \right\} = \frac{2\{r(2-r)-\sigma\}}{\sigma+2r(r-1)}. \tag{20}$$

One leaf replaces one core vertex and contributes $1/p-1=1/(r-1)$ under Bd and -1 under dB.

Both bracketed corrections are positive precisely when

$$L(r,\sigma)<\lambda<U(r,\sigma),$$

where

$$L=\frac{2(1-\sigma)(r-1)}{1+\sigma(r^2-1)},\qquad U=\frac{2\{r(2-r)-\sigma\}}{\sigma+2r(r-1)}.$$

After multiplication by positive denominators, $L<U$ is equivalent to

$$F_r(\sigma)<0,$$

where

$$F_r(\sigma)=(r-1)\sigma^2+(r^3-4r^2+3r+1)\sigma+r(2r-3).$$

Its minimizing value and minimum are

$$\sigma(r)=\frac{-r^3+4r^2-3r-1}{2(r-1)},\qquad \min_{\sigma}F_r(\sigma)=-\frac{P(r)}{4(r-1)}.$$

Exact Sturm counting gives no root of P in $(1,3/2)$ and exactly one in $(3/2,151/100)$. At that root, σ_* is the double tangency and

$$L(R_{\mathrm{hyb}},\sigma_*)=U(R_{\mathrm{hyb}},\sigma_*)=\lambda_*.$$

For fixed σ_*, L is increasing and U decreasing on $(1,R_{\mathrm{hyb}})$ because

$$\partial_rL=\frac{2(\sigma-1)\{\sigma(r-1)^2-1\}}{\{1+\sigma(r^2-1)\}^2}>0,\qquad \partial_rU=\frac{4r(\sigma-r)}{\{2r(r-1)+\sigma\}^2}<0.$$

Thus

$$L(r,\sigma_*)<\lambda_*<U(r,\sigma_*)\quad(1<r<R_{\mathrm{hyb}}),$$

which proves Theorem 2. Compact-uniformity of every estimate and the definition of e_t preserve the required quantifier order.

Proposition 3 (Class-optimal tangent)**.** *Within the dilute heavy-pair plus hub-leaf leading regime* (17)–(18)*,* R_{hyb} *is the exact largest threshold of an interval beginning at one on which a fixed pair* (σ,λ) *can make both corrections positive.*

Proof. For fixed positive σ, feasibility implies $F_r(\sigma)<0$, and hence $\min_{s\in\mathbb{R}}F_r(s)<0$. At R_{hyb} the unconstrained minimum is zero at the admissible value $\sigma_*>0$. Immediately above R_{hyb} the discriminant is negative, so no positive σ—indeed no real σ—is feasible. Since the fixed pair (σ_*,λ_*) works throughout $(1,R_{\mathrm{hyb}})$, no interval beginning at one can extend farther. □

Remark 4 (Rational alternative). Taking $\sigma=19/137$ and $\lambda=20/27$ gives, exactly at $r=3/2$,

$$B=\frac{232}{17361}>0,\qquad D=\frac{65}{12123}>0.$$

This completely rational parameter choice has threshold

$$R_{\mathbb{Q}}=\frac{5069+12\sqrt{147001}}{6439}=1.50176815223369\ldots>3/2.$$

8 Why the fixed-affine endpoint route cannot work

The normalized endpoint coordinates are

$$x(G) = \frac{\rho_{\mathrm{Bd}}(G, 3/2)}{\rho_{\mathrm{Bd}}(K_n, 3/2)}, \qquad y(G) = \frac{\rho_{\mathrm{dB}}(G, 3/2)}{\rho_{\mathrm{dB}}(K_n, 3/2)}.$$

Consider the order-22 complete-support weighted graph with a class A of order two, a class B of order twenty, internal weights 137 and 1, and every cross edge of weight 1/500. Exact solution of the $S_2 \times S_{20}$ orbit chains gives

$$0.9334 < x < 0.9335, \qquad 1.0336 < y < 1.0337, \qquad \frac{x + 2y}{3} - 1 > \frac{2}{10000}.$$

The affine crossing

$$\theta_0 = \frac{y - 1}{y - x}$$

satisfies $\theta_0 > 0.3355 > 1/3$. Hence any universal inequality

$$\theta x + (1 - \theta) y \leq 1$$

would require $\theta > 1/3$. Independently proved clique–pendant asymptotics force every universal coefficient to satisfy $\theta \leq 1/3$ [2]. Therefore no graph-independent convex affine endpoint separator exists. All signs here are exact rational signs from two independent 61-state solves; the displayed decimals are only shorthand.

This refutation is logically separate from Theorem 2. The order-22 graph itself has $x < 1 < y$ and is not a simultaneous endpoint amplifier. The dilute hybrid supplies the endpoint and beyond-endpoint simultaneous family.

9 Independent verification and status

The replay package has three exact components.

1. A labelled-event implementation enumerates all 512 configurations of a nine-vertex test graph and matches every Bd and dB transition aggregate across all 108 orbit fibres.
2. A separate symbolic calculation reconstructs Z_B, Z_D, the pair and leaf correction coefficients, the rational endpoint margins, and the algebraic phase polynomial.
3. An independent Sturm certificate isolates R_{hyb}, verifies the quadratic identity, tangency, optimizer, and derivative factorizations.

The positive-cut diagonal is defined by an exact finite interval decision, not by sampled fitnesses. Numerical sparse solves were used only as a hostile diagnostic for the quantitative core error and are not part of the proof.

The present status is:

PROVED:	$R_{\mathrm{sim}} \geq R_{\mathrm{hyb}} > 3/2$.
PROVED:	R_{hyb} is class-optimal for the displayed dilute regime.
EXACTLY REFUTED:	$R_{\mathrm{sim}} = 3/2$ and every graph-independent convex affine endpoint separator of the displayed form.
OPEN:	The unrestricted exact value of R_{sim} and any matching universal upper bound.

Data and code availability

The source package includes a one-command replay of every finite symbolic component and the independent labelled-chain implementation. The manuscript is a public-repository research draft, but has no tagged preprint release or archival DOI at this checkpoint. A public preprint release would not constitute peer-reviewed journal publication.

AI-assistance disclosure

Computational and language-model assistance was used for symbolic exploration, exact verifier implementation, hostile auditing, and manuscript preparation. Every theorem claim is supported by the displayed analytic argument and independently executable exact certificates. Numerical evidence is explicitly separated from proof.

References

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