

An explicit 44-variable vanishing witness from a 22-variable cubic Keller map

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with heavy assistance from ChatGPT 5.6 Sol

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Abstract

Starting from the recently announced three-dimensional noninjective Keller map, a rank-compressed homogenization of a 13-variable stable model gives a 22-variable cubic homogeneous noninjective Keller map. The de Bondt–van den Essen symmetric reduction then gives a 44-variable homogeneous quartic Hessian-nilpotent polynomial with 538 monomials and an exact collision. Expanded potentials and collisions are supplied as machine-readable exact certificates. For reuse we also record a normalized six-variable gradient lift. A post-release priority audit found that Cassidy had already executed the equivalent six-dimensional symmetric transport; no novelty is claimed for that construction. An appendix preserves the uniform rational collision from the archived first exploration while crediting the earlier, stronger monodromy theorem for the full weighted-lift family.

Verification disclaimer. The author is a complete amateur and cannot independently verify these claims. This is an experiment in the limits of AI-assisted mathematics, not an established result. Every argument, attribution, and novelty claim requires independent expert review. Exact checks are evidence, not peer review.

1 A six-variable symmetric Keller map

Let $i^2 = -1$. The identity-linear normalization of the announced map [2] is

$$\begin{aligned}\Phi_1 &= x - \frac{3}{2}x^2y - \frac{1}{2}x^3z, \\ \Phi_2 &= y + 3x(1 + xy)^2z + 3xy^2(4 + 3xy), \\ \Phi_3 &= (1 + xy)^3z + y^2(1 + xy)(4 + 3xy).\end{aligned}\tag{1}$$

It satisfies $\det J\Phi = 1$ and maps

$$p_0 = (0, 0, -\tfrac{1}{4}), \quad p_1 = (1, -\tfrac{3}{2}, \tfrac{13}{2}), \quad p_2 = (-1, \tfrac{3}{2}, \tfrac{13}{2})\tag{2}$$

to $q = (0, 0, -\tfrac{1}{4})$. For $A, B \in \mathbb{C}^3$, put

$$X = A + iB, \quad \Lambda = \frac{A - iB}{2}, \quad \boxed{\mathcal{S}(A, B) = \Lambda^T \Phi(X)}.\tag{3}$$

Theorem 1. *The gradient map $\nabla \mathcal{S} : \mathbb{C}^6 \rightarrow \mathbb{C}^6$ has symmetric Jacobian, identity linear part, and Jacobian determinant one. The three distinct points*

$$z_j = (p_j/2, -ip_j/2) \in \mathbb{Q}(i)^6 \quad (4)$$

all map to $(q/2, -iq/2) = (0, 0, -1/8, 0, 0, i/8)$. The potential $\mathcal{S}$ has degree eight and 204 monomials when expanded.

Proof. For $f(X, \Lambda) = \Lambda^T \Phi(X)$,

$$\text{Hess}(f) = \begin{pmatrix} \sum_k \Lambda_k \text{Hess}(\Phi_k) & J\Phi^T \\ J\Phi & 0 \end{pmatrix}. \quad (5)$$

Swapping the two blocks of three columns makes this matrix block upper triangular, so $\det \text{Hess}(f) = (-1)^3 (\det J\Phi)^2 = -1$. Equation (3) is the pullback of f by

$$C = \begin{pmatrix} I & iI \\ \frac{1}{2}I & -\frac{i}{2}I \end{pmatrix}, \quad \det C = i. \quad (6)$$

Hessians transform by congruence, giving $\det \text{Hess}(\mathcal{S}) = (\det C)^2 (-1) = 1$. Since $J\Phi(0) = I$, direct multiplication gives $\text{Hess}(\mathcal{S})(0) = I_6$. Finally, $\nabla f(p_j, 0) = (0, q)$; applying C^T proves the collision. $\square$

This is Meng's classical gradient lift [4, 5], specialized to the new map and normalized over $\mathbb{Q}(i)$.

2 Rank-compressed homogenization

Lemma 2. *Let*

$$\Psi(X) = X + H_2(X) + H_3(X) : \mathbb{C}^n \rightarrow \mathbb{C}^n, \quad \det J\Psi = 1, \quad (7)$$

where H_d is homogeneous of degree d . Suppose $H_3 = BK$ for a constant $n \times r$ matrix B and a cubic homogeneous $K : \mathbb{C}^n \rightarrow \mathbb{C}^r$. Define

$$h(X, U, t) = (tH_2(X) + t^2BU, -K(X), 0). \quad (8)$$

Then h is cubic homogeneous and Jh is nilpotent. If $p \neq q$ and $\Psi(p) = \Psi(q)$, then $(p, K(p), 1)$ and $(q, K(q), 1)$ collide under $W \mapsto W + h(W)$.

Proof. For an indeterminate s , a Schur complement in $I + sJh$ gives

$$\begin{aligned} \det(I + sJh) &= \det(I + stJH_2 + s^2t^2BJK) \\ &= \det J\Psi(stX) = 1. \end{aligned} \quad (9)$$

Thus the characteristic polynomial of Jh is T^{n+r+1} . At $t = 1$,

$$(X, K(X), 1) + h(X, K(X), 1) = (\Psi(X), 0, 1), \quad (10)$$

which proves the collision statement. $\square$

The standard homogenization [1] takes $r = n$. Lemma 2 retains only the constant coefficient span of the cubic components.

3 The 22-variable cubic map

The factor-reusing stable reduction in [10] produces

$$\Psi = X + H_2 + H_3 : \mathbb{C}^{13} \rightarrow \mathbb{C}^{13}, \quad \det J\Psi = 1, \quad (11)$$

in the variable order

$$X = (x, y, z, a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, a_5, b_6). \quad (12)$$

The six triangular degree-reduction operations and final output shear are reproduced in the human-readable source accompanying this PDF. They lift the collision (2) to

$$\begin{aligned} \pi_0 &= (0, 0, -\frac{1}{4}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0), \\ \pi_1 &= (1, -\frac{3}{2}, \frac{13}{2}, -1, -2, \frac{9}{2}, -10, \frac{3}{2}, \frac{3}{4}, -\frac{9}{4}, -6, -\frac{27}{8}, \frac{9}{2}). \end{aligned} \quad (13)$$

Exact coefficient row reduction shows that the thirteen components of H_3 span an eight-dimensional space. Take $K = (K_1, \dots, K_8)$ with

$$\begin{aligned} K_1 &= \frac{1}{2}x(a_1z + b_1x), \\ K_2 &= a_2a_3z - 3a_2y^2 + 3b_2a_3x + b_3xy + 12xy^2, \\ K_3 &= -b_1a_3a_4 + a_3b_4y - 3a_4xz + a_5xz - b_6xy + 3xyz, \\ K_4 &= 3x^2y, \\ K_5 &= 2b_1a_3a_4 - 2a_3b_4y + 6a_4xz - 2a_5xz + 2b_6xy - 5xyz, \\ K_6 &= xy^2, \\ K_7 &= -b_1xy - 7a_2a_3z + 21a_2y^2 - 21b_2a_3x - a_3xz - 7b_3xy - 84xy^2, \\ K_8 &= a_4xy. \end{aligned} \quad (14)$$

Then $H_3 = BK$, where

$$B(U_1, \dots, U_8) = (U_1, U_2, U_3, 0, -3U_2, U_4, U_5, 0, 0, U_6, U_7, U_8, 0). \quad (15)$$

The verifier checks (15) coefficient by coefficient and checks $\text{rank } B = 8$. Lemma 2 therefore gives a 22-variable map with 72 cubic monomials across 21 active nonlinear coordinates. Its collision points are $(\pi_j, K(\pi_j), 1)$, where

$$K(\pi_0) = 0, \quad K(\pi_1) = (-\frac{17}{4}, -\frac{45}{8}, \frac{99}{16}, -\frac{9}{2}, -\frac{177}{8}, \frac{9}{4}, \frac{213}{8}, \frac{27}{8}). \quad (16)$$

4 The 44-variable quartic

For the 22-variable map h , define

$$\boxed{\mathcal{P}(A, B) = i \sum_{j=1}^{22} h_j(A + iB)B_j.} \quad (17)$$

The de Bondt–van den Essen characteristic-polynomial identity [3] implies that $\text{Hess}(\mathcal{P})$ is nilpotent exactly when Jh is nilpotent. Hence

$$\Gamma(Z) = Z - \nabla \mathcal{P}(Z) : \mathbb{C}^{44} \rightarrow \mathbb{C}^{44} \quad (18)$$

has Jacobian determinant one.

The cubic collision transports explicitly. If r_0, r_1 collide under $x \mapsto x + h(x)$, let $M_j = I + Jh(r_j)^T$, set

$$y_1 = 0, \quad y_0 = M_0^{-1}(-ih(r_1) + ih(r_0)), \quad (19)$$

and define $z_j = (r_j - iy_j, y_j)$. Direct differentiation gives $\Gamma(z_0) = \Gamma(z_1)$. The two 44-tuples are in `output/collision.json`. Their coordinates are in $\mathbb{Q}(i)$, with numerator at most 261 in absolute value and denominator at most 16.

Theorem 3. *The polynomial $\mathcal{P}$ is a homogeneous quartic in 44 variables with 538 monomials and nilpotent Hessian. The map $Z - \nabla \mathcal{P}(Z)$ is noninjective. Moreover,*

$$\Delta^m \mathcal{P}^m = 0 \quad (m \geq 1), \quad \Delta^m \mathcal{P}^{m+1} \neq 0 \quad (20)$$

for infinitely many m .

Proof. Only the last assertion remains. Zhao [6] proved that Hessian nilpotence is equivalent to $\Delta^m \mathcal{P}^m = 0$ for all $m \geq 1$, and that the formal inverse of $Z - t\nabla \mathcal{P}$ is governed by

$$Q_t = \sum_{m \geq 0} \frac{t^m}{2^m m! (m+1)!} \Delta^m \mathcal{P}^{m+1}. \quad (21)$$

Eventual vanishing would make the inverse polynomial, contradicting the explicit collision. $\square$

5 Certificates, scope, and priority

The expanded files and SHA-256 hashes are

file	SHA-256
<code>symmetric_potential_sparse.json</code>	<code>1e0c97e1c4965c3e...498015c3c3f5fdd4</code>
<code>symmetric_collision.json</code>	<code>6b5b546f24e839a1...89936a711addd0</code>
<code>potential_sparse.json</code>	<code>2a91272816188884...2a65e0751789f4</code>
<code>collision.json</code>	<code>aeab7adb021c07de...89936a711addd0</code>

The full hashes are in `MANIFEST.md`. The symbolic verifier checks the Hessian congruence, the rank-eight factorization, identity (9), exact nilpotence guards, and both collisions. A dependency-free checker reads only the JSON, differentiates both sparse potentials, and verifies the six-variable identity linear part and both finite fibers. A separate Node.js checker independently implements exact rational complex arithmetic with JavaScript `BigInt` and repeats the exported-certificate checks. Neither certificate checker is a stand-alone proof of Hessian nilpotence.

The six-variable result is an explicit instance of Meng’s known reduction, not a new reduction theorem. Cassidy’s public repository commit [9], authored at 14:46:10 UTC on 20 July 2026, had already executed the equivalent de Bondt–van den Essen/Meng transport in $\mathbb{C}^6$, checked symmetry, and transported the same three-point fiber. Our determinant-one, identity-linear formula is a normalized presentation, not an independent discovery. The first version of this note failed to find that artifact and overstated this part of its novelty claim.

Zhang [7] left the Vanishing Conjecture consequence existential. Our earlier note [10] gave a 54-variable certificate. That note is now an archival derivation subsumed by the present paper. Thompson [8] posted a 24-variable cubic homogeneous reduction at 03:29:42 UTC on 21 July 2026. His work contains the same rank-compression principle in executed form and has priority for that

idea. Thompson’s map has 24 variables, 54 cubic monomials, and 23 active nonlinear coordinates; ours has 22 variables, 72 monomials, and 21 active nonlinear coordinates. Both are over $\mathbb{Q}$ with rational collisions. Thus ours is smaller only in ambient dimension, while Thompson’s is sparser. The narrower candidate contribution is the different $13+8+1 = 22$ construction and its 44-variable quartic.

Searches of arXiv, MathOverflow, and public GitHub code on 21 July 2026 found no earlier public 22-variable cubic certificate or executed 44-variable quartic certificate. This is not an exhaustive claim of worldwide priority. The literature is changing by the hour and all novelty statements are provisional.

A A uniform rational collision in a weighted-lift specialization

This appendix preserves the only potentially new explicit certificate from Exploration 01. It is not used in the 22- or 44-variable constructions above. Gallagher introduced the weighted-lift family [11], and Mikhail Szh and Fable [12] publicly proved full S_n monodromy for the entire family before Exploration 01 appeared. No monodromy or deck-group novelty is claimed here.

For $n \geq 3$, put

$$\begin{aligned} u &= 1 + xy, & \gamma &= 1 - \frac{n}{n-1}xy + x^2z, & w &= u\gamma, \\ p_n(w) &= \frac{2w - nw^{n-1}}{n-2}, & q_n(w) &= \frac{w^2 - (n-1)w^n}{n-2}, \end{aligned}$$

and define

$$F_n = \left(\frac{u + q_n(w)/\gamma^2}{x^2}, \frac{1 + p_n(w)/\gamma}{x}, x\gamma \right). \quad (22)$$

The apparent denominators cancel and $\det JF_n = 1$. Let

$$s_n = \frac{4 - 2^n}{n-2}, \quad g_1 = \frac{n+2-2^n}{n-2}, \quad g_2 = 2^{n-1}.$$

For $r \in \{1, 2\}$, let g_r denote the corresponding displayed value and set

$$X_{n,r} = \left(\frac{1}{g_r}, r - g_r, g_r^2 \left[g_r - 1 + \frac{n}{n-1} \left(\frac{r}{g_r} - 1 \right) \right] \right). \quad (23)$$

Both g_1 and g_2 are nonzero for every $n \geq 3$, and direct exact substitution gives

$$F_n(X_{n,1}) = F_n(X_{n,2}) = (s_n, s_n, 1). \quad (24)$$

For example,

$$F_3(-1/3, 4, -54) = F_3(1/4, -2, 36) = (-4, -4, 1).$$

The archived Exploration 01 checker verifies polynomiality, the determinant, and (24) in exact rational arithmetic. This formula is a compact reusable certificate inside the broader Gallagher–Mikhail family, not an independent theorem.

AI-assistance and verification disclosure

The constructions, searches, proofs, programs, website, and drafts were developed with heavy assistance from ChatGPT 5.6 Sol. Alec Kriebel is a complete amateur and cannot independently verify the claims. The note has not been peer reviewed; independent expert scrutiny is essential.

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