

A 14-variable unipotent Keller map and every-order image and vanishing obstructions

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with heavy assistance from ChatGPT 5.6 Sol

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Abstract

Starting from the recently announced three-variable counterexample to the Jacobian conjecture, we give one self-contained consequence mechanism built from a formal inverse pencil and a linear observable. Its first realization is an exact rational vector $g \in \mathbb{Q}[Z]^{14}$ with 24 monomials such that $\det(I + sJg) = 1$, Jg is generically regular nilpotent, and $I + g$ has an exact reduced fiber of three rational points. The same inverse series gives an explicit every-exponent failure of the Special Image Conjecture SIC(14). After homogenization and symmetric reduction it gives a homogeneous degree-eight Hessian-nilpotent polynomial in 30 variables for which $\Delta^m P^{m+1} \neq 0$ at every order. A complementary rank-compressed cubic route gives a homogeneous quartic in 44 variables with the same every-order nonvanishing, directly addressing the classical quartic Vanishing Conjecture. The real 14-variable map also supplies direct counterexamples to the Campbell, Chamberland–Meisters, and Kulikov injectivity and fixed-point formulations. These are one consequence cascade, not independent discoveries. All claims are accompanied by exact sparse certificates and bounded-memory checkers.

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1 The unified result

Let $E_n : \mathbb{C}[\xi, Z] \rightarrow \mathbb{C}[Z]$ be the linear map

$$E_n(\xi^\alpha p(Z)) = \partial_Z^\alpha p(Z), \quad (1)$$

and let Δ_N denote the Laplacian in N variables. The paper proves the following package directly; none of the operator statements is inferred only from a logical equivalence with the Jacobian conjecture.

Theorem 1 (Master theorem). *There are explicit exact objects with the following properties.*

1. A vector $g \in \mathbb{Q}[Z]^{14}$ with 24 monomials satisfies

$$\det(I + sJg) = 1, \quad (Jg)^{14} = 0, \quad (Jg)^{13} \neq 0.$$

The map $T = I + g$ has generic degree three, geometric monodromy S_3 , and an exact scheme-theoretic fiber consisting of three reduced rational points.

2. For $A = -\xi \cdot g$ and $b = x + y + u_{11}$,

$$E_{14}(A^m) = 0, \quad E_{14}(bA^m) \neq 0 \quad (m \geq 1).$$

Thus SIC(14) is false, with one fixed multiplier obstructing every positive exponent.

3. There is a 28-variable nonhomogeneous Hessian-nilpotent polynomial P_{28} of degree eight and a 30-variable homogeneous Hessian-nilpotent polynomial P_{30} of degree eight such that, for $P = P_{28}, P_{30}$,

$$\Delta^m P^m = 0 \quad (m \geq 1), \quad \Delta^m P^{m+1} \neq 0 \quad (m \geq 0).$$

The expanded term counts are 178 and 608, respectively.

4. There is a 44-variable homogeneous Hessian-nilpotent quartic P_{44} , with 538 expanded monomials, such that

$$\Delta_{44}^m P_{44}^m = 0 \quad (m \geq 1), \quad \Delta_{44}^m P_{44}^{m+1} \neq 0 \quad (m \geq 0).$$

The 30- and 44-variable endpoints are complementary: the former has smaller ambient dimension, while the latter lies in the degree-four homogeneous case used in Zhao's equivalence with the Jacobian conjecture [10]. No global minimality claim is made for either.

2 The source map and one inverse series

The following map is the announced three-dimensional counterexample that initiated the present constructions [1]. Put $u = 1 + xy$ and define

$$\begin{aligned} F_1 &= u^3 z + y^2 u(4 + 3xy), \\ F_2 &= y + 3xu^2 z + 3xy^2(4 + 3xy), \\ F_3 &= 2x - 3x^2 y - x^3 z. \end{aligned} \tag{2}$$

We use the identity-linear normalization

$$\Phi = (F_3/2, F_2, F_1). \tag{3}$$

Direct differentiation gives $\det J\Phi = 1$. The three points

$$r_0 = (0, 0, -\tfrac{1}{4}), \quad r_+ = (1, -\tfrac{3}{2}, \tfrac{13}{2}), \quad r_- = (-1, \tfrac{3}{2}, \tfrac{13}{2}) \tag{4}$$

map to $(0, 0, -1/4)$.

The common one-variable engine is the unique series $\tau \in t\mathbb{Q}[[t]]$ satisfying

$$\tau + t\tau^3 = \frac{t}{2}. \tag{5}$$

Let $D = 1 + 3t\tau^2$. Differentiating (5) gives $t\tau' = \tau/D$. Set

$$\bar{x} = t\tau' = \frac{\tau}{D}, \quad \bar{y} = -3t\tau, \quad \bar{z} = tD(1 - 30t\tau^2 - 18t^2\tau^4). \tag{6}$$

Lemma 2 (Exact inverse ray). *The series in (6) satisfy*

$$\Phi(\bar{x}, \bar{y}, \bar{z}) = (t/2, 0, t).$$

Proof. Here $1 + \bar{x}\bar{y} = 1/D$. Direct substitution into (2) gives $F_1 = t$, $F_2 = 0$, and $F_3 = 2\tau(1 + t\tau^2) = t$, the last equality being (5). $\square$

Two linear observables below produce the series

$$q(t) = \tau' - 3\tau - \frac{\tau' - 1/2}{t} = \sum_{m \geq 0} q_m t^m, \quad (7)$$

$$r(t) = \tau' - 3\tau + 3t\tau\tau' = \sum_{m \geq 0} r_m t^m. \quad (8)$$

Lemma 3 (Coefficient-complete obstruction). *For every $k \geq 0$,*

$$\begin{aligned} q_{3k} &= r_{3k} = (-1)^k \frac{\binom{3k+1}{k}}{2^{2k+1}}, \\ q_{3k+1} &= r_{3k+1} = (-1)^{k+1} \frac{3 \binom{3k+1}{k}}{(3k+1)2^{2k+1}}, \\ q_{3k+2} &= (-1)^k \frac{\binom{3k+4}{k+1}}{2^{2k+3}}, \end{aligned} \quad (9)$$

$$r_{3k+2} = (-1)^k \frac{3 \binom{3k+2}{k}}{2^{2k+2}}. \quad (10)$$

In particular, $q_m r_m \neq 0$ for every $m \geq 0$.

Proof. Lagrange inversion in (5) gives

$$[t^{3k+1}]\tau = (-1)^k \frac{\binom{3k+1}{k}}{(3k+1)2^{2k+1}}.$$

Differentiation gives the residue class 0, multiplication by -3 gives class 1, and the remaining summands in (7) and (8) give class 2. These classes are disjoint; elementary binomial simplification gives (9)–(10). $\square$

3 Two inverse-series transfer principles

The next two identities are the conceptual spine of the paper.

3.1 The image-operator transfer

Let $H \in \mathbb{C}[Z]^n$, suppose $\det(I + tJH) = 1$, and let Q_t be the formal inverse of $I + tH$. Put

$$A_H(\xi, Z) = - \sum_{j=1}^n \xi_j H_j(Z). \quad (11)$$

Lemma 4 (Abhyankar–Gurjar transfer). *For every polynomial p ,*

$$p(Q_t(Z)) = \sum_{m \geq 0} \frac{t^m}{m!} E_n(pA_H^m)(Z). \quad (12)$$

Consequently $E_n(A_H^m) = 0$ for every $m \geq 1$, and any nonzero coefficient $[t^m]p(Q_t)(Z_)$ proves $E_n(pA_H^m)(Z_*) \neq 0$.*

Proof. Apply the Abhyankar–Gurjar inversion formula [2, 8] to $I + tH = I - (-tH)$. The determinant factor is one, and the multinomial theorem collects terms of total ξ -degree m . Taking $p = 1$ gives the first conclusion. $\square$

3.2 The symmetric Laplacian transfer

Suppose additionally that $H(0) = 0$. For $A, B \in \mathbb{C}^n$, define the de Bondt–van den Essen symmetric transform [5]

$$P_H(A, B) = i \sum_{j=1}^n H_j(A + iB)B_j, \quad \Gamma_t = I - t\nabla P_H. \quad (13)$$

Lemma 5 (Symmetric transfer). *Write $X = A + iB$. In coordinates (X, B) ,*

$$\Gamma_t(X, B) = (X + tH(X), (I + tJH(X)^T)B - itH(X)). \quad (14)$$

Hence

$$\det J\Gamma_t = \det(I + tJH)^2 = 1,$$

so P_H is Hessian nilpotent. Moreover, the $A + iB$ component of Γ_t^{-1} is $(I + tH)^{-1}$.

Let λ be a linear form on $\mathbb{C}^n$, put $\ell(A, B) = \lambda(A + iB)$, let v be its coefficient vector in the $2n$ variables, and write $D_v = v \cdot \nabla$. If

$$\lambda((I + tH)^{-1}(Y_*)) = \sum_{k \geq 0} c_k t^k,$$

then

$$D_v(\Delta_{2n}^m P_H^{m+1})(Y_*, 0) = 2^m m! (m+1)! c_{m+1}. \quad (15)$$

Proof. Differentiating (13) gives

$$\nabla_A P_H = iJH(X)^T B, \quad \nabla_B P_H = -JH(X)^T B + iH(X),$$

which proves (14). The determinant identity for every t is equivalent to nilpotence of $\text{Hess } P_H$.

Because $H(0) = 0$, the polynomial P_H has order at least two. Zhao's inversion formula [10, Theorem 3.4] for a Hessian-nilpotent polynomial therefore writes

$$\Gamma_t^{-1}(Z) = Z + t\nabla R_t(Z), \quad R_t = \sum_{m \geq 0} \frac{t^m}{2^m m! (m+1)!} \Delta_{2n}^m P_H^{m+1}. \quad (16)$$

Apply ℓ at $(Y_*, 0)$ and compare the coefficient of t^{m+1} . □

Zhao also proved that Hessian nilpotence is equivalent to $\Delta^m P^m = 0$ for every $m \geq 1$. Thus a coefficient-complete inverse observable gives a coefficient-complete counterexample to the corresponding Vanishing Conjecture without ever expanding a high Laplacian power [10, 9].

4 The 14-variable regular-nilpotent map

Write

$$\Phi = X + H_2 + H_3 + \cdots + H_7. \quad (17)$$

The nonzero homogeneous pieces of $\Phi - I$ are

d	$H_{d,1}$	$H_{d,2}$	$H_{d,3}$
2	0	$3xz$	$4y^2$
3	$-\frac{3}{2}x^2y$	$12xy^2$	$3xyz$
4	$-\frac{1}{2}x^3z$	$6x^2yz$	$7xy^3$
5	0	$9x^2y^3$	$3x^2y^2z$
6	0	$3x^3y^2z$	$3x^2y^4$
7	0	0	x^3y^3z

Take state coordinates

$$U = (u_{11}, u_{12}; u_{21}, u_{22}, u_{23}, u_{24}; u_{31}, u_{32}, u_{33}, u_{34}, u_{35}).$$

Let $B \in \text{Mat}_{3 \times 11}(\mathbb{Q})$ select the three chain heads, so $BU = (u_{11}, u_{21}, u_{31})^T$, let $N = J_2 \oplus J_4 \oplus J_5$ with ones immediately above the diagonal, and put

$$C = (H_{3,1}, H_{4,1}; H_{3,2}, H_{4,2}, H_{5,2}, H_{6,2}; H_{3,3}, H_{4,3}, H_{5,3}, H_{6,3}, H_{7,3})^T.$$

Define

$$g(X, U) = (H_2(X) + BU, -C(X) - NU). \quad (18)$$

In the coordinate order $Z = (x, y, z, U)$, this is

$$\begin{aligned}
g_1 &= u_{11}, & g_2 &= 3xz + u_{21}, & g_3 &= 4y^2 + u_{31}, \\
g_4 &= \frac{3}{2}x^2y - u_{12}, & g_5 &= \frac{1}{2}x^3z, \\
g_6 &= -12xy^2 - u_{22}, & g_7 &= -6x^2yz - u_{23}, \\
g_8 &= -9x^2y^3 - u_{24}, & g_9 &= -3x^3y^2z, \\
g_{10} &= -3xyz - u_{32}, & g_{11} &= -7xy^3 - u_{33}, \\
g_{12} &= -3x^2y^2z - u_{34}, & g_{13} &= -3x^2y^4 - u_{35}, \\
g_{14} &= -x^3y^3z.
\end{aligned} \quad (19)$$

Theorem 6. *The vector (19) has 24 monomials and*

$$\det(I + sJg) = 1.$$

Its generic nilpotency index is 14. The fiber of $T = I + g$ over $c = (0, 0, -1/4, 0, \dots, 0)$ consists scheme-theoretically of exactly three reduced rational points, obtained by lifting (4). The generic degree is three and the geometric monodromy is S_3 .

Proof. The Jacobian pencil has block form

$$I + sJg = \begin{pmatrix} I_3 + sJH_2 & sB \\ -sJC & I_{11} - sN \end{pmatrix}.$$

Since $N^5 = 0$, a Schur complement gives

$$\begin{aligned}
\det(I + sJg) &= \det(I_3 + sJH_2 + s^2B(I - sN)^{-1}JC) \\
&= \det\left(I_3 + \sum_{d=2}^7 s^{d-1}JH_d(X)\right) = \det J\Phi(sX) = 1.
\end{aligned}$$

Cayley–Hamilton gives $(Jg)^{14} = 0$, while exact multiplication gives

$$((Jg)^{13})_{1,5} = -3x^6y^4z \neq 0.$$

On the stated target the state equations uniquely give $U = (I - N)^{-1}C(X)$, and the first block becomes $\Phi(X)$. The source-fiber ideal has lexicographic Gröbner basis

$$4z + 1 - 27x^2, \quad 2y + 3x, \quad x(x - 1)(x + 1). \quad (20)$$

Its quotient is reduced of length three, proving the exact fiber assertion.

More generally, over a target (Y, V) the state block gives

$$U = (I - N)^{-1}(V + C(X)), \quad \Phi(X) = Y - B(I - N)^{-1}V.$$

Thus T is stably equivalent to $\Phi \times I_{11}$ over the target space, not merely on the displayed special fiber.

For completeness, a target (a, b, c) of the unnormalized map F has inverse parameter t satisfying

$$2at^3 - bt^2 + 2t - c = 0,$$

with

$$y = -\frac{bt^2 + 3c - 6t}{2t^2}, \quad x = \frac{t}{1 - ty}, \quad z = \frac{2x - 3x^2y - c}{x^3}.$$

Thus the generic degree is at most three; the reduced three-point fiber and étaleness give the reverse inequality. The cubic discriminant is

$$-4(27a^2c^2 - 18abc + 16a + b^3c - b^2),$$

and the discriminant in c of the parenthesized quadratic is $-(12a - b^2)^3 \neq 0$. It is therefore squarefree in $\mathbb{C}(a, b)[c]$ and not a square in $\mathbb{C}(a, b, c)$. Hence the transitive degree-three geometric monodromy group is S_3 . $\square$

The weights

$$\omega = (1, 1, 1; 2, 3; 2, 3, 4, 5; 2, 3, 4, 5, 6) \quad (21)$$

satisfy $\text{wt}(g_i) = \omega_i + 1$. If $D_s = \text{diag}(s^{\omega_i})$, then

$$D_s^{-1}TD_s = I + sg \quad (s \neq 0). \quad (22)$$

Thus the entire punctured pencil is linearly conjugate. One diagonal conjugate has integer coefficients; separately, the member $I + g/4$ has an entirely integral three-point collision and common target $(0, 0, -1, 0, \dots, 0)$. These are two different normal forms.

5 Every-exponent SIC and the named consequence cascade

Let

$$A = -\sum_{j=1}^{14} \xi_j g_j, \quad b = x + y + u_{11}, \quad Z_* = (\tfrac{1}{2}, 0, 1, 0, \dots, 0). \quad (23)$$

For the inverse Q_t of $I + tg$, state elimination and Lemma 2 give

$$Q_{t,x}(Z_*) = \tau', \quad Q_{t,y}(Z_*) = -3\tau, \quad Q_{t,u_{11}}(Z_*) = -\frac{\tau' - 1/2}{t}. \quad (24)$$

The last equality is also the first target equation $Q_x + tQ_{u_{11}} = 1/2$. Hence $b(Q_t)(Z_*) = q(t)$.

Theorem 7. *For every $m \geq 1$,*

$$E_{14}(A^m) = 0, \quad E_{14}(bA^m)(Z_*) = m!q_m \neq 0.$$

Consequently $\ker E_{14}$ is not a Mathieu subspace and $\text{SIC}(14)$ is false.

Proof. Apply Lemma 4 with $H = g$ and use Lemma 3. □

Zhao proved [11]

$$\ker E_{14} = \sum_{j=1}^{14} (\xi_j - \partial_{Z_j}) \mathbb{C}[\xi, Z].$$

Following Liu–Sun’s terminology [7], the assertion that this image is a Mathieu subspace is the Special Image Conjecture. Thus Theorem 7 also refutes Zhao’s regular-sequence Image Conjecture, of which SIC is this specialization. It does not, by itself, reverse other one-way implications in the literature.

The same map gives the following direct real and complex corollaries. The conjecture numbers in the first column are Campbell’s [3]; the other formulations are due to Chamberland–Meisters [4] and Kulikov [6].

Statement refuted	Map	Exact obstruction
Campbell Conj. 3 (unipotent univalence)	T on $\mathbb{R}^{14}$	$\text{Spec } JT = \{1\}$ and three real preimages
Chamberland–Meisters Conj. 2.1	T on $\mathbb{R}^{14}$	all eigenvalues have modulus one, hence a uniform gap from zero
Campbell Conj. 4 (stability / weak injectivity)	$-T$	spectrum constantly $\{-1\}$ and noninjectivity
Kulikov $JN(14)$ and Campbell Conj. 6	$N_c = c - g$	nilpotent derivative and exactly three fixed points
Campbell Conj. 5 (fixed-point form)	translate N_c at p_0	the origin and two other fixed points; spectrum $\{0\}$

These statements are one transformation family, not five independent discoveries. The classical dynamical Markus–Yamabe problems already had older counterexamples; the present maps only give unusually rigid constant-spectrum realizations.

Padding g by zero coordinates proves the corresponding map, fixed-point, and $\text{SIC}(n)$ failures for every $n \geq 14$. The generic Jordan type becomes $(14, 1^{n-14})$.

6 The 28- and 30-variable Hessian companions

Directly apply (13) to $H = g$ and call the result P_{28} . It is nonhomogeneous of degrees two through eight and has 178 expanded monomials. At $(Z_*, 0)$, take

$$\ell_{28}(A, B) = b(A + iB).$$

Lemma 5 and (24) give

$$D_{v_{28}}(\Delta_{28}^m P_{28}^{m+1})(Z_*, 0) = 2^m m!(m+1)!q_{m+1} \neq 0. \tag{25}$$

For the homogeneous version introduce w and put

$$h(Z, w) = (w^7 g(Z/w), 0) \in \mathbb{Q}[Z, w]^{15}. \quad (26)$$

It is homogeneous of degree seven and has 24 monomials.

Proposition 8. *The matrix Jh has exact generic nilpotency index 14 and Jordan type $(14, 1)$.*

Proof. Write

$$Jh = \begin{pmatrix} W & q \\ 0 & 0 \end{pmatrix}, \quad W = w^6 Jg(Z/w), \quad q = w^6 (7g - Jg(Z/w)(Z/w)).$$

Besides $(Jg)^{14} = 0$, exact sparse multiplication gives $(Jg)^{13}g = 0$. Hence $W^{14} = 0$ and $W^{13}q = 0$, so $(Jh)^{14} = 0$. On the other hand,

$$((Jh)^{13})_{1,5} = -3w^{67}x^6y^4z \neq 0.$$

The only partition of 15 with largest part 14 is $(14, 1)$. $\square$

Let $P_{30} = P_h$ as in (13). This polynomial is homogeneous of degree eight and has 608 expanded monomials. Since the last coordinate of $I + th$ is w , its inverse on the slice $w = 1$ is exactly the inverse of $I + tg$. With target $(Z_*, 1; 0)$ and $\ell_{30}(A, B) = b(A + iB)$, Lemma 5 gives

$$D_{v_{30}}(\Delta_{30}^m P_{30}^{m+1})(Z_*, 1; 0) = 2^m m!(m+1)!q_{m+1} \neq 0. \quad (27)$$

Equations (25) and (27), together with Hessian nilpotence, prove parts (3) of Theorem 1.

7 The every-order homogeneous quartic

We now retain degree four at the cost of a larger ambient dimension. A factor-reusing stable degree reduction in the classical framework [2] gives

$$\Psi = X + H_2 + H_3 : \mathbb{C}^{13} \rightarrow \mathbb{C}^{13}, \quad \det J\Psi = 1, \quad (28)$$

in variables

$$X = (x, y, z, g1a, g1b, g2a, g2b, g3a, g3b, g4a, g4b, g5a, g6b).$$

For reproducibility, the six stable operations are summarized below. A two-factor step appends outputs $a + P, b + Q$ and subtracts the indicated coefficient times their product from the target coordinate; one-factor steps reuse the named existing output.

step	target	new or reused factors	coefficient
1	Φ_1	$x^2, xz + 3y$; add $g1a, g1b$	$-1/2$
2	Φ_2	$3x^2y, 2z + xyz + 3y^2$; add $g2a, g2b$	1
3	Φ_2	$xy, g2a z + 3xg2b$; add $g3a, g3b$	-1
4	Φ_3	$xy^2, 7y + 3xz + 3xy^2 + x^2yz$; add $g4a, g4b$	1
5	Φ_3	add $g5a + g4a xy$; reuse $g1b + xz + 3y$	-1
6	Φ_3	reuse $g3a + xy$; add $g6b + g4a g1b - yg4b$	1

A final output shear subtracts the product of the $g3a$ - and $g1b$ -outputs from the $g4b$ -output. Each displayed step is a composition with triangular source and target automorphisms, so it preserves the constant Jacobian up to its linear normalization. Let M be the resulting map and $L = JM(0)$, so $\Psi = L^{-1}M$ and $\det J\Psi = 1$. Exact coefficient reduction factors $H_3 = BK$ with $K = (K_1, \dots, K_8)$ and

$$\begin{aligned}
K_1 &= \tfrac{1}{2}x(g1a\,z + g1b\,x), \\
K_2 &= g2a\,g3a\,z - 3g2a\,y^2 + 3g2b\,g3a\,x + g3b\,xy + 12xy^2, \\
K_3 &= -g1b\,g3a\,g4a + g3a\,g4b\,y - 3g4a\,xz + g5a\,xz - g6b\,xy + 3xyz, \\
K_4 &= 3x^2y, \\
K_5 &= 2g1b\,g3a\,g4a - 2g3a\,g4b\,y + 6g4a\,xz - 2g5a\,xz + 2g6b\,xy - 5xyz, \\
K_6 &= xy^2, \\
K_7 &= -g1b\,xy - 7g2a\,g3a\,z + 21g2a\,y^2 - 21g2b\,g3a\,x - g3a\,xz - 7g3b\,xy - 84xy^2, \\
K_8 &= g4a\,xy.
\end{aligned} \tag{29}$$

Here

$$B(U_1, \dots, U_8) = (U_1, U_2, U_3, 0, -3U_2, U_4, U_5, 0, 0, U_6, U_7, U_8, 0). \tag{30}$$

Define the cubic homogeneous vector in 22 variables

$$h_4(X, U, w) = (wH_2(X) + w^2BU, -K(X), 0). \tag{31}$$

A Schur complement gives

$$\det(I + sJh_4) = \det(I + swJH_2 + s^2w^2BJK) = \det J\Psi(swX) = 1;$$

hence Jh_4 is nilpotent. The checker verifies this matrix identity directly.

Set

$$Y = L^{-1}(\tfrac{1}{2}, 0, 1, 0, \dots, 0) = (\tfrac{1}{2}, 0, 1, 0, 0, 0, -2, 0, \dots, 0) \in \mathbb{Q}^{13}, \tag{32}$$

and $R_* = (Y, 0^8, 1) \in \mathbb{Q}^{22}$. If $X(t)$ is the first block of $(I + th_4)^{-1}(R_*)$, the second block gives $U = tK(X)$ and therefore

$$\Psi(tX) = tY, \quad M(tX) = t(\tfrac{1}{2}, 0, 1, 0, \dots, 0).$$

The stable outputs and Lemma 2 yield

$$X_x = \tau', \quad X_y = -3\tau, \quad X_{g3a} = 3t\tau\tau'. \tag{33}$$

Indeed the zero $g3a$ -output is $tX_{g3a} + (tX_x)(tX_y) = 0$. Thus the linear observable $x + y + g3a$ equals $r(t)$ on this inverse.

Finally define

$$P_{44}(A, B) = i \sum_{j=1}^{22} h_{4,j}(A + iB)B_j. \tag{34}$$

It is a homogeneous quartic with 538 expanded monomials. At $(R_*, 0)$ let $\ell_{44}(A, B) = (x + y + g3a)(A + iB)$. Lemmas 5 and 3 give

$$D_{v_{44}}(\Delta_{44}^m P_{44}^{m+1})(R_*, 0) = 2^m m!(m+1)!r_{m+1} \neq 0 \quad (m \geq 0). \tag{35}$$

This proves the quartic part of Theorem 1 by a closed formula at every order, strengthening the earlier collision-based conclusion of nonvanishing at infinitely many unspecified orders. This is the homogeneous-quartic formulation equivalent to the Jacobian conjecture [10]; the degree-eight companion instead addresses the general homogeneous Vanishing Conjecture [9].

8 Verification, scope, and archival policy

The unified certificate contains the fully expanded 178-term and 608-term potentials, the fixed inverse targets, the two coefficient formulas, the fiber basis, and hashes of the precursor 14- and 44-variable sparse certificates. The primary exact checker verifies the block identities, weighted conjugacy, exact fiber, $(Jg)^{13}g = 0$, both inverse series, symmetrization gradients, degrees, and term counts. Its peak memory in the reference run was below 100 MB. A dependency-free standard-library checker audits the exported sparse data and the first 1000 nonzero coefficients. Finite coefficient checks are guards; the all-order proof is Lemma 3.

The formula $3 + (4 - 2) + (6 - 2) + (7 - 2) = 14$, the stable degree reductions, and the symmetric reduction are classical. The Discovery 06 precursor contains a coefficient-Hankel proof of optimality only inside its stated constant-state realization ansatz; that scoped side result is not needed here. No global smallest-dimension or smallest-term assertion is made. Zihan Zhang had already recorded existential Image and quartic Vanishing consequences of the announced counterexample [12]; the contribution sought here is an explicit fixed-dimensional, every-order realization. At the audit cutoff, no earlier public artifact was located with this exact 14-variable map, an explicit $\text{SIC}(n)$ witness for $n \leq 14$, or the every-order 44-variable quartic formula. That is provisional search evidence, not proof of worldwide priority.

This paper is the canonical consequence paper. The earlier 22/44-variable note, the 21-variable SIC note, and the 14-variable construction note remain public as timestamped technical precursors and certificate sources; they are not presented as three additional publications. The separate iterated monodromy paper studies different mathematics and is not absorbed here.

AI assistance and responsibility

The constructions, audits, programs, certificates, website, and manuscript were developed with heavy assistance from ChatGPT 5.6 Sol under Alec Kriebel’s direction. Alec Kriebel is the named human author and has requested prominent disclosure that he is not qualified to validate the mathematics independently. Independent expert scrutiny is essential.

References

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